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THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

The Association of Mathematics Teachers of India (AMTI) was started in 1965 for the promotion of efforts to improve Mathematics education at all levels. A major aim of the Association is to assist practising teachers of Mathematics in schools in improving their expertise and professional skills. Another important aim is to spot out and foster Mathematical talents in the young. The Association also seeks to disseminate new trends in Mathematics education among parents and public. Other activities of the Association include consultancy services to schools in equipping the Mathematics section of their libraries, in organising children's Mathematics clubs and fairs, in setting up teacher centres in schools, in conducting Mathematics laboratory programmes, in holding practical tests in Mathematics in assisting children in participating investigational projects.

The Association holds "National Mathematical Talent Search Competition" annually and organises Orientation Courses, Seminars and Workshops for teachers and courses for talented students. A national conference is held annually in different parts of the country for teachers to meet and deliberate on important issues of Mathematics education. Innovative teacher award has been instituted to give public recognition to enterprising and pioneering teachers of Mathematics for which entries from teachers are invited.

An award for contributions to the Mathematics Teacher relating to History of Mathematics in the context of mathematics education has been instituted by Prof. R.C. Gupta.

"The Mathematics Teacher (India)" (MT) is the official quarterly journal of the Association and is issued twice a year. It has been approved for use in schools and colleges of education by the Government departments of education in many States. Besides MT the Association also brings out Junior Mathematician (JM), three issues in a year, especially for school students in English and Tamil.

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The Mathematics Teacher

(INDIA)

OFFICIAL JOURNAL OF THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

Volume: 57 Issues: 1 & 2 Year: 2021

Meghraj Bhatt

Editor

EDITORIAL

In July, 2022, I was given the responsibility of editing this journal of AMTI, starting from Vol. 57. This is the first issue i.e. Vol.57 (1&2). As usual, it contains NMTC question papers and solutions. Some equations, notations, subscripts, superscripts were not passing through mail in word file as it should be, there is a delay in editing the manuscript. The AMTI office and Dr. Hemlathaji had to mail me several times, but ultimately it is done. We welcome articles on alternative methods of solving some problems – if there are - and on new problems that can be formed based upon these. I am thankful to The AMTI EC members for giving me this opportunity. I shall try to complete the task to the best of my abilities.

---- Meghraj Bhatt

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SCREENING TEST - GAUSS CONTEST
NMTC at PRIMARY LEVEL
V & VI STANDARDS

1. If the length of a rectangle is doubled and the breadth becomes a third of the original, then the area of the rectangle

   A. is Doubled B. Becomes one third
   C. Becomes two thirds D. None of these.

   Solution:

   If 'a' and 'b' units are the original length and breadth, then the area is 'ab' square units. The new length and breadth are '2a' and 'b/3' units, respectively. Then the new area is 2/3 (ab) square units. Thus, the area of the rectangle becomes two thirds of the original.
   Answer: C

2. Among the following numbers, which one is not the number of diagonals in a polygon of n sides?

   A. 20 B. 28 C. 35 D. 14

   Solution:

   In a polygon of n sides the number of diagonals is given by n(n-3)/2 since each vertex can be connected to n-3 vertices to form diagonals and each diagonal is counted twice, once from each end. 20 = 8 x 5 /2, 14 = 7 x 4/2, 35 = 10 x 7 /2. 28 cannot be written thus.
   Answer : B

3. In a triangle ABC, AB = 10, BC = 5 and AC = 7. If the sides AB and AC are doubled keeping BC the same, then area of the triangle ABC
   A. Remains same. B. Doubles
   C. Becomes four times D. None of these

Solution:

The new AB = 20, AC = 14 and BC = 5 (remains same). Here AB > AC + BC. Triangle inequality is violated. They do not form a triangle.
Answer: D

4. A number can be written in base 3, using four digits but ***cannot*** be written in base 4, using three digits. (47 can be written in base 4 as 233 and as 1202 in base 3, using 3 digits in base 4 and 4 digits in base 3) The number of such numbers is?

A. 17 B. 63 C. 80 D. 143

Solution:

In base 3, the valid digits are 0,1 & 2 and so four digit numbers will be in the range of 1000 to 2222 which are equivalent to 27 to 80 in decimal system. Similarly in base 4, the valid digits are 0,1,2 & 3 and so three digit numbers are in the range from 100 to 333 which are equivalent to 16 to 63 in decimal system.
The numbers required in the problem can be written in base 3 i.e. from 27 to 80 but the numbers that cannot be written in base will be more than 63 i.e. from 64 to 80. So total such numbers are 17.
Answer: A

5. The sum of all the divisors of n, including n and 1, is 24. Then the number of such n is

A. 0 B. 1 C. 2 D. 3

Solution:

If n has 3 distinct prime factors, the least being, 2, 3, 5, the sum of the divisors will be > 1 + 2 + 3 + 5 + 6 + 10 + 15 > 24. So, n can have at the most 2 distinct prime factors. Clearly n cannot

be a power of a prime, but it can be prime. So, n = 23 fits the bill with sum of the divisors = 24. If it has two prime factors say, 2, 3, then if n = 6, sum of its divisors is 12, if n = 12, sum of its divisors is 1 + 2 + 3 + 4 + 6 + 12 = 27. Hence it must have either 2 and 5 or 3 and 5 or 2 and 7. If it has 2 and 5, sum of its divisors is 1 + 2 + 5 + 10 = 18. If it is 3 and 5, then sum of the divisors is 1 + 3 + 5 + 15 = 24 the required value. If it is 2 and 7, then, the sum of the divisors is 1 + 2 + 7 + 14 = 24. We have three solutions n= 14, 15, 23.

Answer: D

6. Positive integers a, b, c with $a \leq b \leq c$, are such that $\frac{7}{16} = \frac{1}{a} + \frac{1}{ab} + \frac{1}{abc}$. Then a + b + c equals

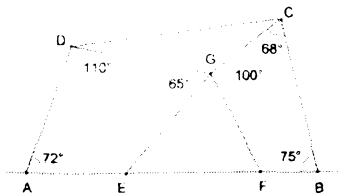
A. 10 B. 11 C. 12 D. None of these

Solution:

One possibility is $\frac{7}{16} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ Hence, a = 4 , b = 2, c = 2. But $a \leq b \leq c$ is not satisfied. If a = 3, then $\frac{1}{ab} + \frac{1}{abc} = \frac{7}{16} - \frac{1}{3} = \frac{5}{48} = \frac{1}{12} + \frac{1}{48}$. Thus, ab = 12 and abc = 48, resulting in b = 4 and c = 4. Hence, a + b + c = 11

Answer: B

7. In the given figure, ABCD, AEGD, and FGCB are quadrilaterals; EFG and CDG are triangles; Points A, E, F, B, all lie on a straight line. The angle measures are shown in the figure. The measure of ∠CGD =



A. 135_0 B. 145_0 C. 155_0 D. 165_0

Solution:

$\angle GEA = 360 - 110 - 65 - 72 = 113^0$. Hence $\angle GEF = 180 - 113 = 67^0$. $\angle GFB = 360 - 100 - 68 - 75 = 117^0$. Hence $\angle GFE = 180 - 117 = 63^0$. From ΔGEF, $\angle EGF = 180 - 63 - 67 = 50^0$. Hence $\angle CGD = 360 - 65 - 50 - 100 = 145^0$.
Answer: B

8. The number of four digit numbers with unit's place 8 and tens place 4 which are divisible by 16 and when the digits are reversed are also divisible by 16 is

A. 0 B. 1 C. 2 D. 3

Solution:

Let the 4 digit number be AB48. If this is divisible by 16, then (AB)x100 is divisible by 16 and hence, AB is a multiple of 4. The reverse 84BA is divisible by 16 implies 84 x 100 + BA is divisible by 16. Thus, BA is divisible by 16. Look at multiples of 16 whose reverses are multiples of 4, which is only 48. If BA is 48, then AB is 84 and the **only** number with the required property is 8448.
Answer: B

9. [ABC] is a three digit number. [BC] is the two digit number. If $[BC] = C^2$ and $[ABC] = [BC]^2$, then the number of divisors of [ABC] including 1 and itself is

A. 3 B. 4 C. 5 D. 6

Solution:

If $[BC] = C^2$ then C = 5 or 6. If C = 5, then [BC] = 25 and $[ABC] = 25^2 = 625$. Number of divisors of $625 = 5^4$ is 5. If C = 6, then $36^2 = 1296$ a 4-digit number. Hence it is invalid.
Answer: C

10. Six boys collected a total of 44 marbles. Each boy collected different natural number of marbles. No boy collected prime number of marbles. Total number of marbles collected by one of the six boys must be

A. 4 B. 9 C. 10 D. 8

Solution:

The least possible marbles that they can have is 1, 4, 6, 8, 9, 10 since they must be non-prime numbers. These add up to 38 only. The excess 6 marbles we have to distribute among these numbers so that they are non-prime and distinct. We can add six marbles to 6, 8, 9, 10 to get 12, 14, 15 and 16 which are composite. This implies it is not necessary that 6, 8, 9 or 10 will be a part of the solution. We cannot add6 to 1 or 4 as it will give 7 and 10, 7 a prime and 10 is repeated. By splitting 6 as 1 + 5 or 2 + 4 or 3 + 3 and adding the components to the numbers 1, 4 we get either prime or already existing numbers. Thus 1 and 4 must be a part of the solution.
Answer: A

11. Two positive numbers are taken. They are replaced with the sum and their difference. The process is repeated three times. Here is an example: (15, 7) (22, 8) (30, 14) (44, 16). Aruna took two positive numbers and got the result (64, 36) after such a process is applied three times. The larger number among the two numbers that Aruna started with is

A. 40 B. 25 C. 32 D. 200

Solution:

Since (64, 36) was obtained as (a + b, a – b) from (a, b), we get a + b = 64 and a – b = 36. This gives a = 50 and b = 14. To get (50, 14) as (a + b, a – b), we must have a + b = 50 and a – b = 14. This leads to a = 32 and b = 18. Finally, to get (32, 18) as (a

+ b, a – b) we must have a + b = 32 and a – b = 18. This results in a = 25 and b = 7. So, the larger number that Aruna started with is 25

Note: Here the "difference" to be taken is "positive difference" in the first step, or we have to suppose that a>b.

Alternative method:
Let the positive numbers be a & b; a>b.
$\therefore$ (a,b) $\rightarrow$(a+b, a-b) $\rightarrow$(2a, 2b) $\rightarrow$ (2a+2b, 2a-2b).

In the data, 2a+2b = 64 & 2a – 2b = 36
i.e. a+b = 32 & a-b=18
$\Rightarrow$ a=25, b=7.
Answer: B

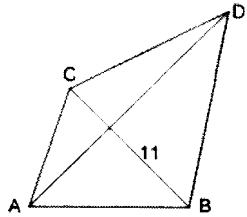
12. In a dodecagon (polygon of 12 sides) the only angles are 90^0 and 270^0. Other angles are not present. What is the number of right angles that it will have?

A. 6 B. 7 C. 8 D. 9

Solution:

The sum of the angles of a dodecagon is 180 x(12 – 2) = 1800^0. Let n be the number of 90^0 angles. Then 90n + 270(12 – n) = 1800 i.e.180n = 1440, giving n = 8.
Answer: C

13. The four vertices and the six edges of a tetrahedron are labelled by the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 11 in such a way that the label on the edge is the sum of the labels of its end vertices. In the figure the edge BC is labelled 11. Then the edge AD is labelled as



A. 8 B. 6 C. 4 D. 3

Solution:

Note that B + C = 11. We have to solve by trial and error method, but with some logic. We can take B+C as 9+2, 8+3, 7+4 or 6+5.

(i) 9+2 will not work because by taking any number at a third vertex, one side will have a number which is not given.

(ii) 8+3 also will not work by the same logic.

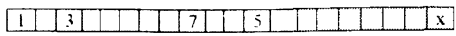
(iii) 7+4 will work because by taking 1&2 for other two vertices A and D, all the sides will get numbers from the given list. So AD=3.

(iv) 5+6 will not work as in (i) & (ii).

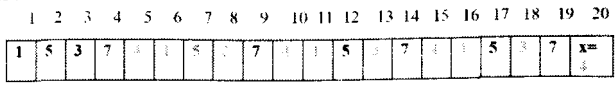
Answer: D

14. 20 numbers are written in twenty cells so that the sum of the numbers in *every* 5 consecutive cells is 20. Four cells are filled with numbers. What is the number in the cell marked x?

A. 5 B. 7 C. 4 D. 3



Solution:



Note that the first, sixth, eleventh and sixteenth positions will be 1.

The third, eighth, thirteenth and eighteenth positions will be 3.

As 9^{th} position is 7, 4^{th} , 14^{th} and 19^{th} positions will be 7.

As the 12^{th} position is 5, 2^{nd} , 7^{th} , 17^{th} positions will be 5.

We are left with four positions only, 5^{th}, 10^{th}, 15^{th}, and 20^{th}. We use the condition that the sum of five consecutive numbers is 20. This results in the 5^{th} 10^{th} , 15^{th} , and 20^{th} numbers to be 4.

Answer: C

15. Let S be the sum of all the four digit numbers that can be formed with the four digits of 2121. The greatest factor of S less than half of S is

A. 909 B. 99 C. 101 D. 3333

Solution:

The six numbers formed are 2112, 2121, 2211, 1122, 1212, 1221. Sum of these is 9999 which has prime factors 3, 11, 101 and other factors too. Greatest factor less than 9999/2 is 3333.

Another way to think:
S = 9999 and the smallest number (except 1) by which S is divisible is 3. We have S= 3 × 3333 so the other factor 3333 is the greatest factor and is less than S/2.
Answer: D

16. a, b, c, d are different natural numbers such that a + 2b = b + 3c = d. The smallest possible value of d is ---------.

A. 7 B. 8 C. 10 D. 11

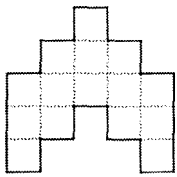
Solution:

We have a + b = 3c, where a, b, c, are distinct. Therefore, c is 2 or more and d = b + 3c is 7 or more. Take a = 5 and b = 1. We get c = 2 and d = 1 + 3 x 2 = 7. Smallest d is 7.

Answer: A

17. The figure below is made up of five triominos vertically and symmetrically placed with perimeter 24 units. If 2021 triominos are put together symmetrically having a zigzag perimeter above and below what will be the perimeter? ---------. (A straight triomino is a rectangle formed by three identical squares, joined side by side.)

A. 8064 B. 8088 C. 8096 D. less than 8000



Solution:

If we add 2 more triominos one on either side, then the perimeter increases by 8. To move from 5 to 2021 triominos we need to add 1008 pairs of triominos. Hence, the perimeter increases by 1008 x 8 = 8064. Hence the perimeter will be 8064 + 24 = 8088.

Answer: B

18. The sum of 30 *not necessarily distinct* positive integers is 901. The least possible LCM of these 30 numbers is

A. 930 B. 31 C. 62 D. 32

Solution:

The least possible LCM will be obtained when you choose 28 numbers equal to 32 and the remaining two numbers are 1 and 4. The LCM is 32. For all other cases, the LCM will be greater.
Answer: D

19. Let N be the largest 7 digit multiple of 7 with different digits. Then the sum of the digits of N is

A. 39 B. 41 C. 40 D. 42

Solution:

The largest number of 7 distinct digits is 9876543. On dividing this by 7,5 remains. Subtracting 5 gives repeated digit. Subtracting 12, gives 9876531 which is divisible by 7 with all distinct digits. So the sum of its digits is 39.
Answer: A.

20. Aruna (1), Bama (2), Chitra (3) are classmates. Only one among the three following statements are true.
(i) Bama is the tallest among the three
(ii) Aruna is not the tallest among the three
(iii) Chitra is not the shortest among the three.
Then the three placed in increasing order of their heights is

A. 123 B. 213 C. 231 D. 132

Solution:

If (i) is true, then ((ii) and (iii) are false. But this implies both Bama and Aruna are the tallest - not possible. So (i) is false. If (ii) is true, then both Bama and Aruna are not the tallest. So Chitra is the tallest and hence (iii) is also true – not possible. Hence (iii) is true and (i) and (ii) are false. Therefore, Aruna is the tallest. Chitra comes in the middle position and Bama is the shortest. Order is 231.
Answer: C

21. Mallika has 108 identical cubes of side 1cm. She makes a cuboid whose base perimeter is 24 cm. Then the maximum possible height of the cuboid is -------

A. 3 B. 4 C. 6 D. 9

Solution:

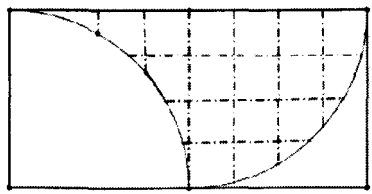
Let a, b be the sides of the base of the cuboid. Then $2(a + b) = 24$. Since $a + b = 12$ and area of the base ab must divide 108, which is the volume, possible values of a, b are 9 and 3 or 6 and 6. In the first case the height is $108/27 = 4$. In the second case the height is $108/36 = 3$. Maximum possible height of the cuboid is 4.

Answer: B

22. In the following rectangle of size 4 x 8 cm, two arcs of a circle are drawn as shown in squares of size 4 x 4. The checked area in cm^2 is ---------

A. 12 B. 16

C. 20 D. Cannot be calculated exactly



Solution:

The area is area of the rectangle – area of a square = 32 – 16 = 16 cm^2. The portion outside the checked quarter circle in the right square is equal to the checked portion outside the quarter circle in the left square. Hence the answer.

Answer: B.

23. When three different natural numbers are multiplied, the result is 40. Their sum can never be

A. 14 B. 11 C. 15 D. 13

Solution:

The factors of 40 are 1,2,4,5,8,10,20,40. The following triplets of different numbers from these gives product 40:
(1,2,20), (1,4,10), (1,5,8), (2,4,5). Their sums are respectively 23, 15, 14 & 11.
So 13 is the answer.
Answer: D

24. Nithya gave birth to a baby boy in the year 1964. She named her child as Rahul. The life period of Nithya was 1940 - 2010 and that of her son was 1964 - 2006. There were occasions when Rahul's age was a factor of Nithya's age. Here are a few examples:

Rahul's Age (in years)	1	2	3	4	6	...
Nithya's Age (in years)	25	26	27	28	30	...

How many such occasions were there (including the given examples)?

A. 6 B. 7 C. 8 D. 9

Solution:

Lifetime of Nithya is 70 and of Rahul is 42. Nithya was 24 when Rahul was born. Hence The possible ages of Rahul and Nithya pair is (1, 25), (2, 26), (3, 27). (4, 28), (6, 30), (8, 32), (12, 36), (24, 48). After this Rahul's age will be more than half of Nithya.
Hence 8 is the answer.
Answer: C

25. The HCF of a 2-digit number y and 40 is 1. How many such 2-digit numbers y are there?

A. 31 B. 45 C. 36 D. 37

Solution:

$40 = 2^3 5$. For HCF of 40 and y to be 1, y must not have the prime factors 2 and 5. Also $10 \leq y \leq 99$. Number of two digit numbers which are multiples of 2 or 5 is $45 + 9 = 54$. Number of two digit numbers remaining is $90 - 54 = 36$. Hence 36 two digit numbers are relatively prime to 40.
Answer: C

26. On a dance floor some boys and girls were there. When 30 girls left, there were two boys for each girl. Sometime later 50 boys leave. Then there were 3 girls for each boy. Then the number of boys in the beginning is ---------

A. 40 B. 45 C. 50 D. 60

Solution 1:

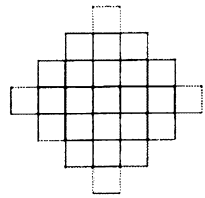
Let g and b be the number of boys and girls in the beginning. After 30 girls leave, $2(g - 30) = b$. After 50 boys leave, $g - 30 = 3(b - 50)$
We have $2g - b = 60$ --- (1)
$3b - g = 120$ --- (2) Adding (1) and (2) we get
$2b + g = 180$ --- (3) Adding (2) and (3) we get
$5b = 300$ i.e., $b = 60$. Hence, $g = (b + 60)/2 = 60$.

Solution 2:

If b boys remain in the end, then, there are 3b girls. In the earlier stage, before the boys left 3b girls and $b + 50$ boys were there. From the given condition $b + 50 = 2 \times 3b = 6b$. Hence, $b = 10$ and initially there were $b + 50 = 60$ boys.
Answer: D

27. In the following figure the small boxes are squares and the total area of the Figure is 225cm^2 . Then the perimeter of the figure in cm is ---------

A. 75 B. 78 C. 84 D. 90



Solution:

Total number of squares 25. Each square has area 225/25 = 9 cm^2. Hence, the side of each square is 3 cm. The perimeter is 28 x 3 = 84 cm.
Answer: C

28. The HCF of two natural numbers is equal to one-fourth of one number and one-fifth of other number. If these two numbers have HCF < 7 and LCM > 100, then the sum of these two numbers is --------.

A. 45 B. 54 C. 60 D. 64

Solution:

Let h be the HCF of the two numbers. Then the two numbers are 4h and 5h. It is given that h < 7 and the lcm of the two numbers 4 x 5 x h = 20h > 100. Then h must be only 6. The two numbers are 24 and 30. Their sum is 54.
Answer: B

29. A digital clock shows the time 14 : 32 (02 : 32 pm). The least time period to pass in order to see again the same four digits 1; 2; 3; 4 (in some order) on the clock will be ---------- minutes.

A. 422 B. 522 C. 431 D. 118

Solution:

It is 21 : 34 (09 : 34 PM). Time between these two is 7 hours and 2 minutes, i.e., 422 minutes.
Answer: A

30. Consecutive even numbers are arranged in rows as follows: The first row has first 2 even numbers 2 and 4; The second row has next 3 even numbers 6, 8 and 10; the third row has next 4 even numbers 12, 14, 16 and 18. Every row thereafter has one more even number than the previous row. No even number is repeated or missed out. The pattern continues with consecutive even number arrangement in this manner. In which row number, the 100^{th} even number appears?

Row 1			2	4		
Row 2		6	8	10		
Row 3	12	14	16	18		
Row 4	20	22	24	26	28	
⋮	⋰		⋮		⋱	

A. 12 B. 15 C. 13 D. 14

Solution:

The number of even numbers in n rows is $2 + 3 + \ldots + (n+1) = n(n+3)/2$. This crosses 100 for n =13. For n = 12, this will be 90. Up to 12^{th} row 90 even numbers will be accommodated. The 13^{th} row will accommodate 14 more. Hence 100^{th} even number will be in row 13.
Answer: C
Note:
The 100^{th} even number is 200. Now correlate the row number with the 1^{st} number in that tow.

This gives the following table:

Row no.	First no.	Relation
1	2	1×2
2	6	2×3
3	12	3×4
4	20	4×5
⋮	⋮	⋮

From this we can judge that the 1^{st} no. in the thirteenth row will be $13 \times 14 = 182$ and in the fourteenth row will be $14 \times 15 = 210$. So 200 will be in the 13^{th} row.

SCREENING TEST - KAPREKAR CONTEST
NMTC at SUB JUNIOR LEVEL
VII & VIII STANDARDS

1. Sadhana's date of birth is d: m: 2008 and Bhavna's date of birth is D: 05: 2008, their date of births given in the (date: month: year) format. D, d, m are all composite numbers and pairwise co-primes. Also, D + d + m = 48. Then, the birth month of Sadhana is

   A. June B. October C. September D. April

   Solution:

   D, d, m cannot be 1, must be ≥ 4, being composite. Possible values for m are 4, 6, 8, 9, 10, 12. Since D and d, are ≤ 30 being composite, they can have prime factors up to 13. If m = 12, then D + d = 36 and both must be odd and cannot have 3 as a prime factor. They must have odd prime factors 5, 7, 11, 13. We can easily eliminate multiples of 13 and 11 and 7 from consideration as the smallest acceptable coprime number will be > 30. If we take D = 25, then d = 11 is a prime. If m = 9, we have D + d = 39. And one of D, d is even. As possible odd factors are 5, 7, 11, 13, D, d can take values 25 and 14 which are both composite and D, d, m are pairwise relatively prime. Hence m = 9, September.
   Answer: C

2. a, b are positive integers such that $a^2 - ab - 20b^2 - 2020 = 0$. Then, the *smallest* possible value of a – 7b is

   A. 2 B. – 8 C. – 222 D. – 101

   Solution:

   $a^2 - ab - 20b^2 = 2020$; (a – 5b)(a + 4b) = 2020 = 1 x 2020, 2 x 1010, 4 x 505, 5 x 404, 10 x 202, 20 x 101. a + 4b > a – 5b as a, b are positive integers. The difference is a +4b – a + 5b = 9b.

The difference in the factors is a multiple of 9. The only possible cases are 1010 – 2 = 1008 and 101 – 20 = 81, which gives 9b = 1008 or 9b = 81 and hence b = 112 or b = 9. The corresponding a = 562 and a = 65. Hence a – 7b = 562 –784 = –222 or a -7b = 65 – 63 = 2.

Answer: C

3. ABC and ABD are isosceles triangles with AB = AC = BD. Also, BD meets AC at P and is perpendicular to AC. Then $\angle C + \angle D$ is

A. 135^0 B. 110^0 C. 120^0 D. 130^0

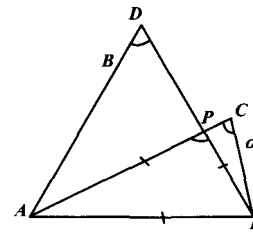
Solution:

P is point of intersection of BD and AC. Let $\angle ACB = \angle ABC = x$. Then $\angle BAC = 180 - 2x$. In right ΔAPB $\angle ABP = 90 - (180 - 2x) = 2x - 90$. Hence $\angle BAD = \angle BDA = (180 - (2x - 90))/2 = 135 - x$. Hence $\angle ACB + \angle BDA = x + 135 - x = 135^0$.

Alternate solution:

Let $\angle C = \alpha$ and $\angle D = \beta$.

From Δ ABC, $\angle ABC = \alpha$ and from Δ ABD, $\angle BAD = \beta$.



Now, $\angle CAB = 180^0 - 2\alpha$

$\therefore \angle DAP = \beta - (180^0 - 2\alpha)$.

Now $\angle DAP + \angle ADP = 90^0$

$\Rightarrow \beta - (180^0 - 2\alpha) + \beta = 90^0$

$\Rightarrow 2\alpha + 2\beta = 270^0$

$\Rightarrow \alpha + \beta = 135^0$

$\therefore \angle C + \angle D = 135^0$

Answer: A

4. The positive integer '*m*' has exactly three positive divisors and the positive integer '*n*' has exactly four positive divisors. The minimum possible number of positive divisors of *m n* is

A. 5 B. 6 C. 7 D. 12

Solution:

The number m must be of the form p^2 where p is a prime since it has three divisors exactly. N must be of the form qr where q and r, are distinct primes or q^3 where q is a prime. For mn to have least number of divisors, m must be of the form p^2 and n must be of the form p^3, where both m and n have the same prime factor. Thus $mn = p^5$ and it has 6 factors. For other forms, $mn = p^2qr$ has 12 factors, $mn = p^2q^3$ has 12 factors, $mn = p^2pq = p^3q$ has 8 factors. Thus, answer is 6.
Answer: B

5. If x = 3, y = 2.3 and z = 3.7 then the value of $\frac{1}{x^2 - xy + yz - zx} + \frac{1}{y^2 - yx - yz + zx} + \frac{1}{z^2 + xy - yz - zx}$ is __.

A. 1 B. 1.47 C. 2.45 D. 0

Solution:

x = 3, y = 2.3 and z = 3.7.
$x^2 - xy + yz - zx = (x - y)(x - z) = 0.7 \times -0.7 = -0.49$
$y^2 - xy - yz + zx = (y - x)(y - z) = -0.7 \times -1.4 = 0.98$
$z^2 + xy - yz - zx = (z - x)(z - y) = 0.7 \times 1.4 = 0.98$
Required sum is 0.
Answer: D

6. Let $a < b < c < d$ be positive integers each being less than 10. How many possible combinations of a, b, c, d are there such that LCM(a, d) = LCM(b, c)?

A. 1 B. 2 C. 3 D. 4

Solution:
The trivial possible solution is
a=1, b=2, c=3, d=6 ---------- (1)

For any other value of d>6, it is not possible to get proper b & c. Again changing c>3 or b>2, d is not possible. Hence (i) is the only solution.

a = 1, b = 2, c = 3, d = 6. One solution only.
Answer: A

7. For which of the following values of k the following equation is not satisfied?

$$\frac{x-3}{x-4}=\frac{x-k}{x-6}$$

A. 3 B. 4 C. 5 D. 7

Solution:

$\frac{x-3}{x-4}=\frac{x-K}{x-6}=\frac{K-3}{2}$ (by compo. – divi.)
$\frac{x-3}{x-4}\neq 1$ for any X. So $\frac{K-3}{2}\neq 1$. i.e. $K\neq 5$

Answer: C

8. In ΔPQR, PQ = PR and S is a point on PR such that QS⊥PR. Given that PS and SR are of integer length in cm and $QS^2 = 63$ cm^2, the least value of SR in cm is

A. 7 B. 1 C. 3 D. none of these

Solution:

$PQ^2 - PS^2 = 63 = PR^2 - PS^2 = (PR - PS)(PQ + PS) = SR \times (PQ + PS)$. SR is the smallest positive integer factor of 63. Hence SR is 1.
Answer: B

9. Five friends Ahmed, Baskar, Charlie, Dravid, Easwar went to a lodge `Stay Safe' to seek room(s). The lodge manager Rahul said that there are only 3 rooms vacant of which two of them are double rooms - Room 143 and Room 145 and the other a single room - Room 131. The friends agreed to take up the three rooms and paid the advance amount for the same. [note: Exactly 2 persons can stay in a double room and Exactly 1 person can stay in a single room]

   Here are few possibilities of 5 friends occupying the 3 rooms:

Serial number	Occupying *Room numbers*		
(Possibility)	143	145	131
1	A, B	C, D	E
2	A, C	B, D	E
3	C, D	A, B	E
4	E, B	D, A	C
⋮	⋮	⋮	⋮

Code: A – Ahmed; B – Baskar; C – Charlie; D – Dravid; E - Easwar

   In how many ways can the friends occupy the 3 rooms (including the above cases)?

   A. 60 B. 30 C. 15 D. 120

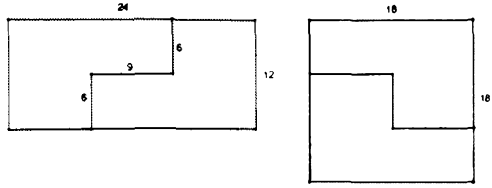
   Solution:

   If E occupies the single room, then the other two double rooms will be occupied by the remaining 4 people in 6 ways (AB, CD; AC, BD; AD, BC; and their reversals). For each of the 5 people, we have 6 such distributions. Hence totally 6 x 5 = 30 ways they can occupy the rooms.
   Answer: B

10. A rectangle of length 27 cm and breadth 12 cm is dissected into two congruent hexagons. These two hexagons can be

reassembled to form a square. Then, the perimeter of each hexagon is --------- cm.

A. 60 B. 56 C. 48 D. 45



Solution:

The above figure shows the way to split the rectangle into congruent hexagons. The perimeter is 18 + 12 + 9 + 6 + 9 + 6 = 60.

Answer: A

11. y is a 3-digit number such that it is equal to 16 times the sum of its digits. Total number of such 3-digit numbers y is

A. 0 B. 1 C. 2 D. 3

Solution:

y = ABC say, where A, B, C are its 3 digits. Given that 100A + 10B + C = 16(A + B + C), we have 84A – 6B – 15C = 0.
Hence 28A = 2B + 5C. C must be even.
Hence $C \leq 8$ and $B \leq 9$. Thus, the RHS ≤ 58.
Therefore A = 1 or 2 only.
$\therefore$ A=1, B=4, C=4 or A=1, B=9, C=2 ----- 2 solutions.
A = 2; C = 8 B = 8 ----- 1 solution
Total 3 solutions.

Answer: D

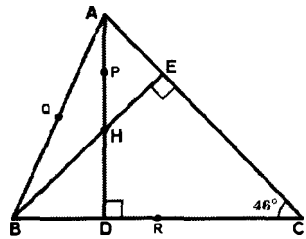
12. Find the number of *integer* pairs (a, b) such that ab – 3a + 4b = 0.

A. 4 B. 6 C. 8 D. 12

Solution:

$ab - 3a + 4b = 0$ will give us $(a + 4)(b - 3) = -12$, where a, b are integers. -12 can be factored as 1 x -12, -1 x 12, 2 x -6, -2 x 6, 3 x -4, -3 x 4 and their reverses -12 x 1, 12 x -1, -6 x 2, 6 x -2, -4 x 3, 4 x -3. Each of these 12 factorizations will give a pair of integer values for (a, b). Hence 12 solutions.
Answer: D

13. In the adjoining figure $\angle C = 46^0$, AD and BE, are the altitudes cutting at H. P, Q, R, are respectively the mid points of HA, AB and BC. Then the measure of the angle PQR is



A. 92^0 B. 98^0 C. 94^0 D. None of these

Solution:

PQ is parallel to BE and QR is parallel to AC. Since BE is perpendicular to AC, $\angle PQR = 90^0$.
Answer: D

14. In how many ways 60 can be divided into 10 different natural numbers?

A. 8 B. 12 C. 10 D. 7

Solution:

The sum of the first 10 different natural numbers is:
$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

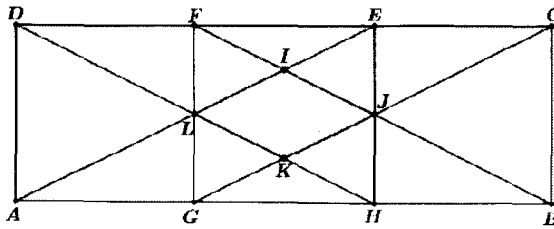
Keeping this as reference, we can increase 5 more on both the sides but the ten numbers on LHS need to be different. These are the following possibilities:

$1+2+3+4+5+6+7+8+9+15=60$
$1+2+3+4+5+6+7+8+14+10=60$
$1+2+3+4+5+6+7+13+9+10=60$
$1+2+3+4+5+6+12+8+9+10=60$
$1+2+3+4+5+11+7+8+9+10=60$
$1+2+3+4+5+6+7+8+13+11=60$
$1+2+3+4+5+6+7+12+9+11=60$

Therefore, the number of ways that 60 can be divided into 6 different natural numbers is 7.
Answer: D.

15. ABCD is a rectangle where AB = 3 cm; BC = 1 cm. G, H, are points on side AB such that AG = GH = HB, as shown. F, E are points on side DC such that CE = EF = FD, as shown. AE, BF intersect at point I and CG, BF intersect at point J, as shown. HD, CG intersect at point K and HD, AE intersect at point L, as shown. Then, the area of the rectangle ABCD is ---- --- times the area of the quadrilateral IJKL.

A. 10 B. 18 C. 12 D. 16



Solution:

Area of ABCD = 3sq.cm. EFGH is a square of area 1 sq. cm. J and L are midpoints of EH and FG. EFLJ is a rectangle and has area ½ sq.cm. Area of Δ LJI = ¼ of rectangle EFLJ = 1/8 sq.

cm. Area of IJKL = 2 area of Δ LJI = ¼ sq. cm. Area of ABCD = 12 times that of IJKL.
Answer: C

16. For how many values of K is 30^{15} the least common multiple of 20^5, 6^8, 15^5, 5^{15} and K?

A. 10 B. 15 C. 20 D. none of these

Solution:

$20^5 = 2^{10}5^5$; $6^8 = 2^8 3^8$; $155 = 3^5 5^5$; $5^{15} = 5^{15}$. LCM = $30^{15} = 2^{15}3^{15}5^{15}$.
Let $K = 2^a 3^b 5^c$. For the LCM of the five numbers to be 30^{15}, a and b both must be 15 and c can be any integer from 0 to 15.
Total value of K is 16.
Answer: D

17. y is a positive integer such that 11 + y; 13 + y; 17 + y; 19 + y are all prime numbers. The *smallest* such y is --------.

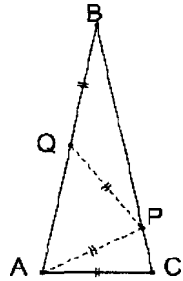
A. 10 B. 90 C. 8 D. 7

Solution:

Note that y must be even and must end in 0. Any other ending will lead to one of these values to end in 5 which will not be a prime. The least such y is 90 as 101, 103, 107,109 are prime.
Answer: B

18. AB = BC in an isosceles triangle ΔABC. Points P, Q are on sides BC,AB respectively such that AC = AP = PQ = QB, as shown. The value of $\frac{\angle BAC + \angle BCA}{\angle ABC}$ is

A. 4 B. 6 C. 8 D. 10



Solution:

Let $\angle ABC = 2x$. Then $\angle QBP = 2x = \angle QPB$ (ΔQBP isosceles). $\angle AQP = 4x$ (exterior angle property in ΔQBP). Hence $\angle QAP = 4x$ and $\angle QPA = 180 - 8x$. Thus, we get $\angle APC = 180 - 2x - (180 - 8x) = 6x = \angle ACP$ (ΔACP is isosceles) $= \angle BAC$ (ΔABC is isosceles). Therefore, $\frac{\angle BAC + \angle BCA}{\angle ABC} = 6$.

Answer: B

19. Let N be the number $x^2 - y^2$, with x = 781,781,781...,7815 having 2020 digits (781 repeating 673 times) and y = 465346534653...46539 having 2021 digits (4653 repeating 505 times). The remainder when N is divided by 22 is ---------.

A. 12 B. 14 C. 16 D. 10

Solution:

Since x and y are both odd, $x_2 - y_2$ is even and hence the remainder must be even. Now, 781 is a multiple of 11.
Therefore 781,781,781...,7810 is a multiple of 2 and 11 and hence 22. Hence, $x = 22m + 5$.
4653 is a multiple of 11 and hence 465346534653...46530 is a multiple of 2 and 11 and hence 22. Hence $y = 22n + 9$.
Thus, $x^2 - y^2 = (22m + 5)^2 - (22n + 9)^2 = 22m(22m + 10) + 25 - 22n(22n + 18) - 81 = 22k + 25 - 81 = 22k - 56 = 22k - 66 + 10 = 22(k - 3) + 10$. Remainder is 10.
Answer: D

20. The sum to 50 terms of the following series $\left[\frac{1}{1}\right]+\left[\frac{1}{2}\right]+\left[\frac{2}{1}\right]+\left[\frac{1}{3}\right]+\left[\frac{2}{2}\right]+\left[\frac{3}{1}\right]+\left[\frac{1}{4}\right]\ldots\ldots$ where [x] is the greatest integer $\leq$ x, is ------- .
(Note that the sum of the numerator and denominator of the fractions are increasing from 2 onwards. For sum two there is one term, for sum three two terms, for sum 4, three terms and so on.)

A. 68 B. 70 C. 71 D. 72

Solution:

Sum of the numerator and denominator increase from 2 onwards. For sum = 2, we have a single term, for 3, two terms, for 4, Three terms and so on. 1 + 2 + 3 +.... + 9 = 45. Terms 46 to 50 have the sum of numerator and denominator as 11. The terms are all of which are 0. The sum is 1 + 2 + 4 + 5 + 8 + 9 + 12 + 14 + 17 = 72
Answer: D

21. Let m and n be *positive integers* such that $2021^2 + m^2 = 2020^2 + n^2$. The sum of all possible values of m + n where m, n satisfy the above equation is

A. 4041 B. 4490 C. 5837 D. 5850

Solution:

$n^2 - m^2 = 2021^2 - 2020^2 = 4041 = (n - m)(n + m)$, $n > m$.
4041 = 1x 4041, 3 x 1347, 9 x 449 where 449 is prime.
n – m = 1, n + m = 4041 gives n = 2021, m = 2020, m+ n = 4041.
n – m = 3, n + m = 1347 gives n = 675, m = 672, m + n = 1347.
n – m = 9, n + m = 449 gives n = 229, m = 220, n + m = 449.
Total = 4041 + 1347 + 449 = 5837
Answer: C

22. F is a simplest fraction whose numerator and denominator are both composite numbers that differ by 9. If $\frac{4}{5} < F < \frac{5}{6} <$, then the largest possible sum of the numerator and denominator of F is

A. 99 B. 81 C. 89 D. 93

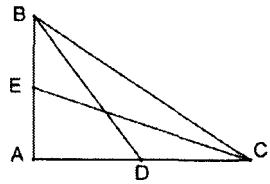
Solution:

Let $F = \frac{a}{a+9}$ where a is a positive integer. We need $\frac{4}{5} < \frac{a}{a+9} < \frac{5}{6}$. This gives two conditions on a. $4a + 36 < 5a$ leading to $a > 36$ and $6a < 5a + 45$, leading to $a < 45$. Both a and a+9 must be composite. Hence $a = 39$ and $a + 9 = 48$, or $a = 40$, $a+9 = 49$. The others have either numerator or denominator as prime! Largest sum of the numerator and denominator of the required F is $40 + 49 = 89$.

Answer: C

23. Let ABC be a right triangle with $\angle A = 90^0$. The length of the medians through B and C are $\sqrt{601}$ and $2\sqrt{61}$. Then AB = ------

A. 10 B. 20 C. 24 D. 25



Solution:

BD = $\sqrt{601}$ and CE = $2\sqrt{61}$. Let AB = c and AC = b. Then $BD^2 = 601$ and $CE^2 = 244$. Solving these equations as $4c^2 + b^2 = 2404$ and $4b^2 + c^2 = 976$, we get $15b^2 = 1500$. Thus, b = 10 and c = AB = 24.

Answer: C

24. The addition given below is incorrect. This can be made correct by changing one digit *d*, wherever it occurs into another digit e. Then d × e = ------.

A. 30 B. 12 C. 20 D. 42

$$\begin{array}{r} 6395427 \\ +\ 5547504 \\ \hline 13044031 \end{array}$$

Solution:

If 5 is replaced with 6 then the addition becomes correct.
Thus d = 5 and e = 6. Hence d x e = 30.
Answer: A

25. What are the last two digits of the *quotient* of 10^{50} when divided by $10^{25} + 7$?

A. 7 B. 49 C. 93 D. None of these

Solution:

$10^{50} - 49 + 49 = (10^{25} + 7)(10^{25} - 7) + 49$. Quotient is $10^{25} - 7$ and remainder 49. Last 2 digits of quotient is 93.
Answer: C

26. A, B, C, D are four numbers given. The sum of a set of three numbers from these are recorded as 1140, 1250, 1335, 955. Then the sum of the largest and smallest among the four numbers is

A. 380 B. 820 C. 2290 D. none of these

Solution:

If a, b, c, d are the four numbers then 3 (a + b + c + d) = 4680. Hence, a + b + c + d = 1560. The largest number is 1560 – 955

= 605 and the smallest is 1560 −1335 = 225. Then the sum of these two is 605 + 225 = 830.
Answer: D

27. Given that x, y, u, v are real numbers such that x + y + u = 4; y + u + v = −5; u + v + x = 0. v + x + y = − 8. Then xyuv = ----.

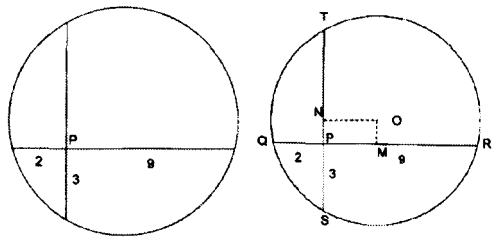
A. 180 B. 210 C. 240 D. 250

Solution:

Adding all the 4 equations we get $3(x+y+u+v) = -9 \Rightarrow x+y+u+v = -3$. Hence $v = -7$, $x = 2$, $y = -3$ and $u = 5$. Therefore xyuv = 210.
Answer: B

28. Two perpendicular chords of a circle intersect at P. The lengths of the parts of the chords are given in the figure. Then double the square of the radius of the circle is ------

A. 32.5 B. 65 C. $2\sqrt{32.5}$ D. $2\sqrt{65}$



Solution:

M and N are the mid points of the chords QR and ST. Hence OM⊥QR and ON⊥ST. QP x PR = SP x PT (Intersection of chords theorem). Hence PT = 6 and NT = 4.5. Thus, MP = MQ − QP = 5.5 − 2 = 3.5 = ON. $OT^2 = ON^2 + NT^2 = 12.25 + 20.25 = 32.5$. Double the square of the radius = 2 x 32.5 = 65 units.
Answer: B

29. ABC is a 3-digit number such that its last digit C is divisible by 2; the two digit part BC is divisible by 3; ABC itself is divisible by 4. How many such 3 digit numbers are there ? ----------.

A. 81 B. 72 C. 63 D. 54

Solution:

ABC itself is divisible by 4 if the two digit part BC is divisible by 4. So, BC is divisible by both 3 and 4 i.e., by 12. C can be only 0, 2, 4, 6, 8.
For C = 0, B is 0 or 6 only
For C = 2, B = 1 or 7 only.
For C = 4, B = 2 or 8 only
For C = 6, B = 3 or 9 only.
For C = 8, B = 4 only.
Totally 9 combinations of BC.
But A can be any non-zero digit.
We have 9 choices. Totally 9 x 9 = 81 choices
Answer: A

30. Let 2499, 3554 and 4398 leave the same remainder "r" when they are divided by d > 1.
Then, the value of (d-r) is_____.

A. 33 B. 178 C. 211 D. None of these

Solution:

2499, 3554 and 4398 leave the same remainder r when divided by d. So, d is a factor of (3554 – 2499) and (4398 – 3554) i.e., 1055 and 844. 1055= 211× 5 and 844 = 211× 4. Note that 211 is prime. As d > 1, d = 211 and r = 178. So, d – r = 33.
Answer: A

SCREENING TEST - BHASKARA CONTEST
NMTC AT JUNIOR LEVEL
IX & X STANDARDS

1. In Δ ABC, D is the midpoint of AC and E is a point on BC such that $\frac{BC}{BE}$ =4. Let BD and AE meet in F. If area of Δ ABF = 1, then area of Δ ABC is

A.3 B. 4 C. 5 D. 6

Solution:

Let CF meet AB in P. By Ceva's theorem, $\frac{AD}{DC}\frac{CE}{EB}\frac{BP}{PA} = \frac{1}{1}\times\frac{3}{1}\times\frac{BP}{PA} = 1$. Hence $\frac{BP}{PA} = \frac{1}{3}$. Hence 4 BP = BA. Since [ABF] = 1, [FBP] = $\frac{1}{4}$. Since D is the midpoint of AC, [ABD] = [CBD], [AFD] = [CFD] and hence [ABF] = [CBF] = 1. Therefore [BCF] = [ABF] = 1. [BCP] = $\frac{5}{4} = \frac{1}{4}$ [ABC]. Therefore, [ABC] = 5.

Answer: C

2. Let x and y be real numbers such that $6x^2 + 2xy + 6y^2 = 9$. Then the maximum value of $x^2 + y^2$ is

A. 5 B. 1 C. 1.4 D. 1.8

Solution:

$5(x^2 + y^2) + (x + y)^2 = 9$. From this we get that $0 \leq 5(x^2 + y^2) \leq 9$. Maximum value of $x^2 + y^2$ is $\frac{9}{5} = 1.8$ when $x + y = 0$. Hence the value is $\frac{9}{5}$.

Answer: D

3. A *cascading number* is one where the digits are strictly decreasing from left to right, like 54321 or 65420. Among all 5 digit cascading numbers from 10000 to 99999 the 202^{nd} number does not have digit

A. 4 B. 5 C. 8 D. 9

Solution:

Smallest cascading number is 43210. If we use digits 0, 1, ..., 8 only then the number of cascading numbers is $\binom{9}{5} = 126$. So, the required number must begin with 9. Using up to 0, 1, ..., 7 for the remaining 4 digits we get $\binom{8}{4} = 70$ numbers. Totally 196 numbers. This means our target number must start with 98. We have 98210, 98310, 98320, 98321, 98410, 98420 as the next 6 numbers. The 202^{nd} number is 98420.
Answer: B

4. A function f from integers to integers is defined as

$$f(n) = n + 5 \text{ if } n \text{ is odd}$$
$$= n/2 \text{ if } n \equiv 2 \pmod 4 \text{ and}$$
$$= n/4 \text{ if } n \equiv 0 \pmod 4.$$

If $f(f(f(n))) = 19$, then the number of such n is

A. 2 B. 3 C. 4 D. more than four

Solution:

If n is odd f(n) is even. Hence f(f(n)) = 38 or 76. If f(f(n)) = 38, then f(n) = 33 or f(n) = 152. When f(n) = 33, n = 66 or 132. If f(n) = 152, n = 147 or 608. If f(f(n)) = 76, then f(n) = 71 or f(n) = 304. If f(n) = 71, n = 142 or 284. If f(n) = 304, n = 299 or 1216.
Hence there are **eight** values for n.
Answer: D

5. If a fraction F is written as 0.282828... in base A and as 0.33333.... in base B then least value of A+B is

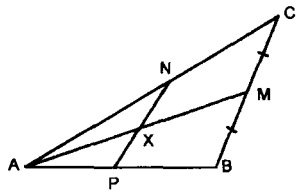
A.17 B. 20 C. 22 D. 24

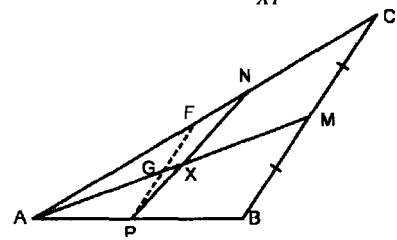
Solution:

$F = \frac{2A+8}{A^2-1} = \frac{3}{B-1}$; i.e. $2(A + 4)(B - 1) = 3(A^2 - 1)$. Hence $A^2 - 1$ is even. A being odd $A^2 - 1$ will be divisible by 8; As A + 4 is also odd B – 1 is divisible by 4. So, B is 5, 9 or 13. A is at least 9 and B must be>A. Hence least value of B is 13. F is ¼ and so $A^2 - 8A - 33 = 0$ resulting in A = 11. Check! A + B = 24.

Answer: D

6. In Δ ABC, M is the midpoint of BC. AB = 20 and AC = 30 units. N and P, are points on AC and AB respectively such that AN = 3AP. NP and AM meet at X. Then $\frac{NX}{XP}$ equals





A. 2 B. 3/4 C. ½ D. 4/3

Solution:

Draw a line through P parallel to BC meeting AM in G and AC in F. AGX is a transversal for Δ PFN. Hence $\frac{NX}{XP} \times \frac{PG}{GF} \times \frac{AF}{AN} = 1$ by Menelaus" theorem. AM being the median, PG = GF. Hence $\frac{NX}{XP} = \frac{AN}{AF} = 3 \times \frac{AP}{AF} = 3 \times \frac{AB}{AC} = 3 \times \frac{20}{30} = 2$.

Answer: A

7. Let $f(x) = A\ sin^3x + B\ tan\ x + Cx^5 - 2$.
Given that $f(\log(log_4 100)) = 5$. Then $f(log\ log\ 2) =$

A. –5 B. 7 C. –7 D. –9

Solution:

Let g(x) = f(x) + 2 = $A\ sin^3x + B\ tan\ x + Cx^5$.

Clearly g(x) is an odd function and $g(log(log_4 100)) = 7$.
$Since, log(log_4 100) = log(2\ log_4 10) = log(\frac{2}{log_{10} 4}) =$
$log(\frac{2}{log_{10} 4}) = log(\frac{2}{2\ log_{10} 2}) = -\ log\ log\ 2.$

Therefore, $g(-log(log\ 2)) = -\ g(log\ (log\ 2)) = g(log(log_4 100)) = 7.$

Hence $g(log\ (log\ 2)) = -7$ and $f(log(log\ 2)) = g(log\ (log\ 2)) - 2 = -9$.

Answer: D

8. How many three digit positive integers are there such that each is a multiple of 11 that contains distinct digits?

A. 24 B. 32 C. 44 D. none of these

Solution:

Let ABC A ≠ 0, be a three digit number divisible by 11. Then there are two possibilities
(i) A+ B = C and
(ii) A + B = 11 + C. We have to count the number of numbers with distinct digits in each case

(i) A + B = C: Since A, B, C are distinct C can have least value 3 only.

C = 3: Then (A, B) = (1, 2) or (2, 1) --- 2 possibilities
C = 4: Then (A, B) = (1, 3) or (3, 1) --- 2 possibilities
C = 5: Then (A, B) = (1, 4), (2, 3), (3, 2), (4,1) --- 4 possibilities
C = 6: Then (A, B) = (1, 5), (2, 4), (4, 2), (5,1) --- 4 possibilities

Similarly, for C = 7 and C = 8

we have 6 possibilities each and for C = 9

we have 8 possibilities. Totally, 32 possibilities for Case

(ii) Since maximum sum of A + B is 18 the maximum value for C is only 7. But for C = 7 we get same value for A and B namely 9.

C = 6: Then (A, B) will be (9, 8), (8, 9) --- 2 possibilities
C = 5: Then (A, B) will be (9,7), (7, 9) --- 2 possibilities
C = 4 and 3 --- 4 possibilities each
C = 2 and 1 --- 6 possibilities each
C = 0: Then (A, B)

will (9, 2), (8, 3), (7, 4), (6, 5), (5, 6), (4, 7), (3, 8), (2, 9)
--- 8 possibilities

Totally, 32 possibilities

Total number of possibilities in both cases is 64.
Answer: D none of these

9. How many prime numbers p are there such that $16p - 27$ is the cube of a positive integer?

A. 0 B. 1 C. 2 D. 3

Solution:

$16p - 3^3 = n^3$; Then, n is odd. $16p = (n + 3)(n^2 - 3n + 9)$. Therefore, $n + 3$ is even and $n^2 - 3n + 9$ is odd. So, $n + 3 = 16$, $n = 13$ and $n^2 - 3n + 9 = 139$ which is prime. Only such prime p is 139.
Answer: B

10. A, B, C are positive integers such that $A.log_{324}(27) + B.log_{324}(8) = C$. The least value of A + B + C is

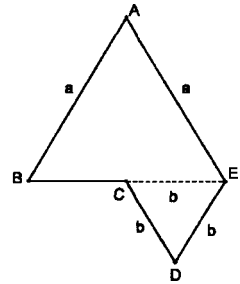
A. 5 B. 7 C. 9 D. 11

Solution:

$A.log_{324}(27) + B.log_{324}(8) = log_{324}(27^A 8^B) = C$.
Hence $27^A 8^B = 3^{3A} 2^{3B} = 324^C = 2^{2C} 3^{4C}$. Hence 3A = 4C and 3B = 2C. Hence 3A = 6B = 4C. The least value of A + B + C is 9 when C = 3, A = 4, B = 2.
Answer: C

11. ABCDE is a concave pentagon that can be dissected into exactly two equilateral triangles with side lengths greater than 1cm. The area of the pentagon is $5\sqrt{3}$ cm^2 and its perimeter is 14 cm. The sum of the lengths of adjacent sides containing the largest vertex angle of the pentagon is

A. 4 cm B. 6 cm C. 3 cm D. 4.4 cm



Solution:

The pentagon will be of the above form with AB = AE = BE = a cm and CD = DE = EC = b cm, thus creating two equilateral triangles. Perimeter is 3a + b = 14 cm and area = $\frac{\sqrt{3}}{4}(a^2 + b^2) = 5\sqrt{3}$ cm^2. Hence $a^2 + b^2 = 20$ and 3a + b = 14. a = 4 or 4.4 cm. This gives b = 2 or 0.8. Only 2 is valid. Then BC + CD = a – b + b = 4 cm.
Answer: A

12. a, b, c are integers such that $ax^2 + bx + c$ is exactly divisible by 5 for every integer x. Then, which of the following is a possible remainder when abc is divided by 1000?

A. 150 B. 200 C. 250 D. 300

Solution:

Let $f(x) = ax^2 + bx + c$. When $x = 0$, $f(0) = c$ is divisible by 5. $f(1) = a + b + c$ and $f(-1) = a - b + c$ are both divisible by 5. Since $f(1) + f(-1) = 2(a + c)$ is divisible by 5 and c is also divisible by 5 we see that a is divisible by 5. Since f(1) is divisible by 5, b is also divisible by 5. Hence abc is divisible by 125. Thus, the remainder when abc is divided by 1000 is a multiple of 125. So, a possible remainder is 250.

Answer: C

13. What is the number of subsets of {1, 2, 3, 4, 5, 6, 7, 8} that are subsets of neither{1, 2, 3, 4, 5} nor {4, 5, 6, 7, 8}?

A.196 B. 192 C. 128 D. 188

Solution:

Total number subsets of {1, 2, 3, 4, 5, 6, 7, 8}is $2^8 = 256$. Number of subsets of {1, 2, 3, 4, 5} are $2^5 = 32$ and the number of subsets of {4, 5, 6, 7, 8} is $2^5 = 32$. But since 4, 5 are common to both the sets, subsets of {4, 5} will be counted in both.

Therefore, total number of required subsets is $256 - 32 - 32 + 4 = 260 - 64 = 196$.

Answer: A

14. Let N = 123456...41424344 be the 79 digit number formed by writing 1, 2, 3, ..., 43, 44 in order, one after the other. The remainder when N is divided by 45 is -----.

A. 4 B. 9 C. 18 D. 27

Solution:

The number leaves a remainder 4 when divided by 5. As the sum of the digits = 4 x 45 + 10 + 20 + 30 + 5 x 4 + 10 = 270 is divisible 9, the number itself is divisible by 9. If N =45m + r, where $0 < r < 45$, then r must be divisible by 9 and must leave a remainder 4 when divided by 5. Possible values for r: 4, 9, 14, 19, 24, 29, 34, 39, 44 which are 4(mod 5).
Answer: B

15. If $f(x) = x^4 + ax^3 + bx^2 + cx + d$ is a polynomial such that $f(1) = f(3) = f(4) = f(6)$, what is the value of b?

A.60 B. −60 C. 67 D. −67

Solution:

Let $f(1) = f(3) = f(4) = f(6) = k$. Then $f(x) - k$ has 1, 3, 4, 6 as roots. Hence, $f(x) - k = (x - 1)(x - 3)(x - 4)(x - 6) = x^4 - 14x^3 + 67x^2 - 126x + 72 = x^4 + ax^3 + bx^2 + cx + d - k$. hence b = 67.
Answer: C

16. In the following figure the areas of the shaded regions are given in cm^2. What is the area of the quadrilateral left out in cm^2?

A.25.5 B. 26.75 C. 26.25 D. 27

[redacted]

Solution:

Join AP. Let area of $\Delta AFP = x$ and area of $\Delta AEP = y$.

$$\frac{AF}{FB} = \frac{[AFP]}{[FPB]} = \frac{x}{4} = \frac{[AFC]}{[FBC]} = \frac{x+y+5}{10}.$$

This gives the equation $6x - 4y = 20$ ---- (1)

$$\frac{AE}{EC} = \frac{[AEP]}{[ECP]} = \frac{y}{5} = \frac{x+y+4}{11}$$

This gives the equation $6y - 5x = 20$ ---- (2)

(1) × 3 + (2) × 2 gives $x = 25/2$.

Hence, $y = 55/4$. $[AFPE] = 105/4 = 26.25\ cm^2$

Answer: C

17. The function f(x) is defined for all real numbers x. If f(a + b) = f(ab) for *all* a, b ∈ ℝ and $f(\frac{1}{2}) = \frac{1}{2}$, then f(2020) is equal to

A. 2020 B. $-\frac{1}{2}$ C. 0 D. $\frac{1}{2}$

Solution:

If f(a + b) = f(ab) ----- (1) for *all* a, b ∈ ℝ
Put a = 0, b = 1; $f(\frac{1}{2}) = f(0) = \frac{1}{2}$. For any a ∈ ℝ and b = 0, $f(a) = f0) = \frac{1}{2}$. Hence $f(2020) = \frac{1}{2}$.
Answer: D

18. A 3 × 5 grid of dots is given as shown. How many straight line ***segments*** can be drawn that join *two of those dots and pass through exactly one other dot*?

A. 14 B. 16 C. 20 D. 22

Solution:

Vertical segments ---- 5
Horizontal segments in each row 3. ∴ Total ----9.
Main diagonal segments --- 2
Slant segments 3 + 3 = 6.
Totally 22 segments.
Answer: D

19. Professor Absentminded has a very peculiar problem, in that he cannot remember numbers larger than 15. However, he tells his wife, "I can remember any number up to 100 remembering the three numbers obtained as remainders when the number is divided by 5, 7 and 11 respectively. For example (2, 3, 6) is 17". Professor remembers that he had (0, 0, 4) rupees in the purse, and he paid (3, 2, 1) rupees to the servant. How much money is left in the purse in Rs.? ---
---

A.47 B. 36 C. 25 D. None of these

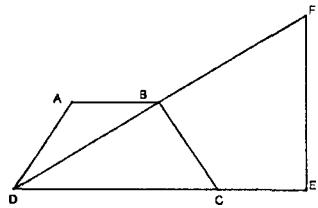
Solution:

(0, 0, 4) means the number is divisible by 5 and 7 and leaves a remainder 4 when divided by 11. So, it is a multiple of 35. It is 70 as it leaves a remainder 4 when divided by 11. (3, 2, 1) means the number leaves a

remainder 3, 2 and 1 when divided by 5, 7, 11. The number must be from 12, 23, 34, 45, 56, 67 as these are < 70 and leave a remainder 1 when divided by 11. It must be 23. Money left in the purse is 47 Rs.

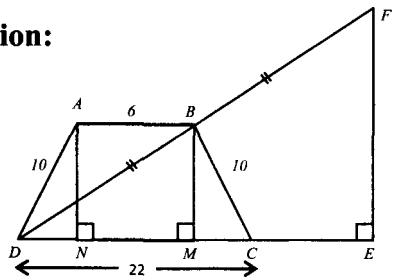
Answer: A

20. ABCD is an isosceles trapezium with AD = BC = 10 cm, AB = 6 cm and DC = 22 cm. E lies on DC extended and FE is perpendicular to DE with D, B, F collinear and B the midpoint of DF. Then CE in cm is



A.4 B. 5 C. 6 D. 7

Solution:



Let BM⊥DC, AN⊥DC.

Then NM=6 and DN = MC= =8.

∴ DM = 22 – 8 = 14 -------- (i)

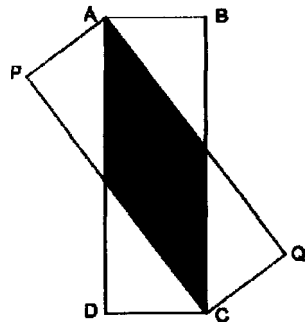
Since B is mid pt of DF and BM || FE,

M is mid pt. of DE ∴ ME=DM=14

∴ CE = ME – MC = 14-8 = 6.

Answer: C

21. Two identical rectangles of size 1 cm x 3 cm are overlapping so that a diagonal of one falls on the diagonal of the other. If S is the common shaded area between the two, then S equals in cm^2



A. 5/2 B. 5/3 C. 5/4 D. 4/3

Solution:

Since the four triangular pieces are congruent in the above figure, the common shaded area is a rhombus of side x say. Then $1^2 + (3 - x)^2 = x^2$ by Pythagoras theorem. Hence $6x = 10$ resulting in $x = 5/3$ cm. If a and b are the diagonals of the rhombus, $a = \sqrt{10}$ and $(b/2)^2 + (a/2)^2 = 25/9$. $(b/2)^2 = 25/9 - 5/2 = 5/18$. Hence, $b = 2\sqrt{5/18}$. Area of the rhombus is ab/2 $= \sqrt{10} \times \sqrt{5/18} = \sqrt{25/9} = 5/3$ cm^2

Answer: B

22. Let x, y, z be real numbers such that $\frac{xy}{x+y} = \frac{1}{3}, \frac{yz}{y+z} = \frac{1}{5}, \frac{zx}{z+x} = \frac{1}{6}$, then the value of $\frac{7xyz}{xy+yz+zx}$ is -----

A. 7 B. 2 C. 1 D. 0.5

Solution:

$\frac{1}{x}+\frac{1}{y}=3, \frac{1}{y}+\frac{1}{z}=5, \frac{1}{z}+\frac{1}{x}=6$. Hence $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=7$.
Now $\frac{7xyz}{xy+yz+zx}=\frac{7}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}=\frac{7}{7}=1$

Answer: C

23. x, y are positive integers such that 101x + 17y = 2020. The value of x + y = ---------.

A.20 B. 104 C. 101 D. No solution exists

Solution:

Look at the solution x = 20, y = 0. This violates the condition x, y > 0. Increase y by 101 and decrease x by 17. 101×(20 – 17) + 17×(0 + 101) = 2020. Hence x = 3 and y = 101 is the only other solution possible as more decrease in X will make x<0. Hence, x + y = 104.
Answer: B

24. An ordered 6 tuple of positive integers (a_1, a_2, a_3, a_4, a_5, a_6) satisfies both the following properties: $a_n + a_{n+1} = a_{n+2}$ for $1 \leq n \leq 4$, $n \in \mathbb{N}$ and $a_5 = 2020$. How many such ordered 6 tuples of positive integers are there ? ----------

A.337 B. 135 C. 336 D. 134

Solution:

Let $a_1 = a$ and $a_2 = b$. Then $a_3 = a + b$, $a_4 = a + 2b$ and $a_5 = 2a + 3b = 2020$ (given).
Hence b is even. Let b = 2k. Hence a + 3k = 1010. As a > 0, k = 1, 2, …, 336.
Answer: C

25. A is a point in the plane of ΔBCD such that AB = AC. Also, $\angle BAC = 2\angle BDC$. AC and BD, meet at E. If AB = 6 and EC = 2 units, then BE×ED = ------ $unit^2$

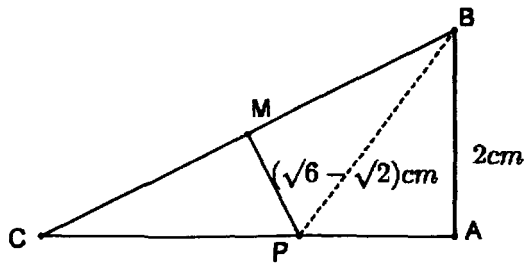
A. 24 B. 20 C. 8 D. None of these

Solution:

With A as centre and AB = AC as radius draw a circle. The circle will pass through D. Since AB = AC = 6, the diameter of the circle is 12 units. Chord BD and the diameter through C meet at E. Hence BE. ED = CE.(12 – CE) = 2 x 10 = 20 $units^2$.

Answer: B

26. ABC is a right triangle where $\angle BAC = 90^\circ$. M, P, are points on sides BC, AC respectively such that MP is the perpendicular bisector of BC, as shown. If AB = 2 cm and MP = $(\sqrt{6} - \sqrt{2})$ cm, then CP =



A. 4 cm B. $(\sqrt{6} - \sqrt{2})$ cm

C. $(2 + \sqrt{3})$ cm D. 3 cm

Solution:

Since MP is the perpendicular bisector, $\angle PCB = \angle PBC = \alpha$. Let BM = x, CP = y = PB and MP = z. Sin $\alpha = \frac{z}{y}$, cos $\alpha = \frac{x}{y}$ (from ΔBMP) and sin $2\alpha = 2 \sin \alpha \cos \alpha = \frac{2}{y} = \frac{2zx}{y^2}$ (from ΔABP). Hence, $zx = y$. This gives, $z^2(y^2 - z^2) = y^2$; $y^2(z^2 - 1) = z^4$; $y^2(7 - 4\sqrt{3}) = 16(2 - \sqrt{3})^2 = 16(7 - 4\sqrt{3})$. Hence, y = 4 cm.

Answer: A

27. Let x and y be real numbers satisfying $2x^2 - 42xy + 441(y + 1)(y - 1) \leq 42x - 882$. Then the value of $4(x^2 + 64y^2)$ is ---------.

A. 2020 B. 2021 C. 2019 D. 1729

Solution:

The inequality reduces to

$2x^2 - 42xy + 441y^2 - 42x + 441 \leq 0.$

$(x - 21y)^2 + (x - 21)^2 \leq 0.$

This is satisfied only if

$x = 21y$

and $x = 21$;

i.e., $y = 1$.

Hence, $4(x^2 + 64y^2) = 4(441 + 64) = 2020$.

Answer: A

28. k is a 4-digit number such that the difference between k and its reversed four digit number is 2088. The number of such 4-digit numbers k, is ---.

A.81 B. 63 C. 126 D. 162

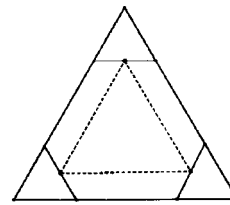
Solution:

Let ABCD be the 4 digit number such that A, D $\neq$ 0. Assume ABCD > DCBA. Hence, ABCD – DCBA = 999A + 90B – 90C – 999D = 2088. 111(A – D) + 10(B – C) = 232. From given conditions, 1 – 9 $\leq$ A – D $\leq$ 9 – 1 and 0 – 9 $\leq$ B – C $\leq$ 9 – 0. Hence A – D = 2 and B – C = 1. Number of choices for (A, D) is 7 and (B, C) is 9. Total number of cases is 63. But we have to count both ABCD and DCBA as we are talking about the difference. So totally we have 126 choices.

Answer: C

29. The picture below shows an equilateral triangle whose sides have length 2020 cm. In each corner of the triangle a smaller equilateral triangle with sides of length 840 cm is inscribed. The midpoints of the inner sides of these three smaller triangles are joined to form an equilateral triangle in the centre of the large equilateral triangle. What is the side length of this new equilateral triangle in cm? --------

A.760 B. 840 C. 680 D. None of these



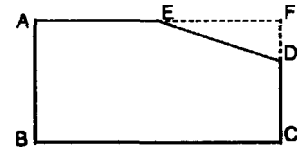
Solution:

There are three isosceles trapeziums in the figure, with the length of the slant sides 420 and the smaller of the parallel sides = 2020 – 1680 = 340 cm. Since the adjacent angles along the slant sides are 60° and 120°, the difference between the larger and smaller parallel sides is 2 x 420 x (1/2) = 420 cm. Hence the side of the inner equilateral triangle is 340 + 420 = 760 cm.

Answer: A

30. A pentagon is formed by cutting off a triangular piece from the corner of a rectangle. The sides of the pentagon in some order are 5, 25, 13, 40 and 37 units. Then the area of the rectangle is --------- square units.

A.1240 B. 1480 C. 925 D. 1000



Solution:

As BC is the largest side of the pentagon, it is clear that BC=40.
Let AB=a, EF=x, FD=y, then AE=40-x,
ED=$\sqrt{x^2 + y^2}$and CD=a – y.

So
a, 40-x, $\sqrt{x^2 + y^2}$and a-y are in some order 5, 25, 13 and 37.
Taking $\sqrt{x^2 + y^2}$=5 or 25, remaining sides cannot be found.
Taking $\sqrt{x^2 + y^2}$=37,

we get
x=35, y=12 or
x=12, y=35

but then a≠5.

Take a=25, CD=25-12=13, AE=40-35=5 will work.

∴ Area = a × 40 = 25 × 40 = 1000 sq. Units.
Answer: D

SCREENING TEST - RAMANUJAN CONTEST
NMTC AT INTER LEVEL
XI & XII STANDARDS

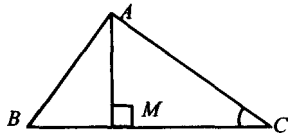
1. The length of the altitude on the side BC of a triangle ABC equals BA – AC. Then $\sin\left(\frac{C-B}{2}\right)+\cos\left(\frac{B+c}{2}\right)$ is equal to

   A. 1 B. 1/2 C. 1/3 D. none of these

Solution:

Given AM = c –b.

Sin C= $\frac{AM}{AC}=\frac{c-b}{b}$



$\therefore$ b sin C = c – b; 2R sin B sin C = 2R sin C – 2R sin B.

Therefore,

$$\cos(B-C)-\cos(B+C)=4\sin\left(\frac{C-B}{2}\right)\cos\left(\frac{B+C}{2}\right);$$

$$1-2\sin^2\left(\frac{B-C}{2}\right)-1-2\cos^2\left(\frac{B-C}{2}\right)=4\sin\left(\frac{C-B}{2}\right)\cos\left(\frac{B+C}{2}\right);$$

$$\sin^2\left(\frac{C-B}{2}\right)+\cos^2\left(\frac{B+C}{2}\right)+2\sin\left(\frac{C-B}{2}\right)\cos\left(\frac{B+C}{2}\right)=0$$

$$\sin\left(\frac{C-B}{2}\right)+\cos\left(\frac{B+C}{2}\right)=0.$$

Answer: D

2. The last 3 digits of $\left[\frac{10^{100}}{10^{20}+5}\right]$, where [x] is the greatest integer $\leq$ x, is

   A. 124 B. 125 C. 625 D. 624

Solution:

$10^{100} = (10^{20})^5$. Take the numerator as $10^{100}+5^5-5^5$. Then,

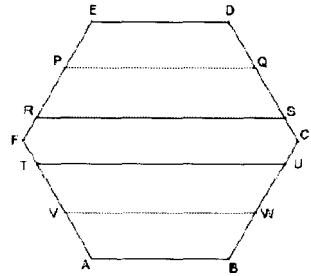
$$\left[\frac{10^{100}}{10^{20}+5}\right] = \frac{10^{100}+5^5}{10^{20}+5} - 1 = 10^{80} - 5\times10^{60} + 5^2\times10^{40} - 5^3\times10^{20} + 5^4 - 1$$

The last 3 digits will be $5^4 - 1 = 624$

Answer: D

3. A regular hexagon ABCDEF is divided into four trapeziums (EDQP; PQSR; TUWV; VWBA) and one hexagon (RSCUTF) by lines parallel to AB as shown. If the trapeziums and the hexagon RSCUTF have the same perimeter, the ratio EP : PR : RF is

A. 8 : 2 : 1 B. 12 : 4 : 1 C. 9 : 3 : 1 D. 6 : 3 : 1



Solution:

Let the side of the given hexagon be 1 unit. Let EP = a, PR = b and RF = c. Then a + b + c = 1. Then AB = 1, PQ = 1 + a, RS = 1 + a + b, CF = 2.

Perimeter of EDQP = 1 + 1+ a + a + a = 2 + 3a.

Perimeter of PQSR = 1 + a + b + b + 1 + a + b = 2 + 2a + 3b = 2 + 3a implies **a = 3b.**

Perimeter of RSCUTF = 1 + a + b + 1 + a + b + 4c = 2 + 2a + 2b + 4c = 2 + 2a + 3b implies **b = 4c.**

So, a = 12c, b = 4c. Hence a : b : c = 12c : 4c : c = 12 : 4 : 1

Answer: B

4. How many integers x are there such that $x(x+1)(x+7)(x+8)$ is a positive integer and a perfect square?

A. 2 B. 3 C. 4 D. 5

Solution:

Let $x^2 + 8x = y$. Then, $x(x+1)(x+7)(x+8) = y(y+7) = n^2$ a perfect square. $2y(2y+14) = (2n)^2 = (2y+7)^2 - 7^2$. This leads to $(2y + 2n + 7)(2y - 2n + 7) = 49$.
As $n > 0$ and y is an integer, we have the following cases:

(1): $2y + 2n + 7 = 49$
$2y - 2n + 7 = 1$. This gives $2y + 7 = 25$, $4n = 48$,
i.e., $y = 9$ and $n = 12$.
$x^2 + 8x = 9$ gives $x = -9$, $x = 1$

(2): $2y + 2n + 7 = -1$
$2y - 2n + 7 = -49$.
This gives $2y + 7 = -25$, $4n = 48$,
i.e., $y = -16$ and $n = 12$.
$x^2 + 8x = -16$ gives $x = -4$.
Hence there are 3 solutions.

Answer: B

5. Given that x, y satisfy the equation $x^2 + y^2 = 18x + 8y + 3$, the maximum value for $5x + 12y$ is

A. 93 B. 223
C. 37 D. data insufficient to determine

Solution:

$x^2 + y^2 = 18x + 8y + 3$
$\therefore (x^2 - 18x + 81) + (y^2 - 8y + 16) = 3 + 16 + 81$
$\therefore (x - 9)^2 + (y - 4)^2 = 81 + 16 + 3 = 100 = 10^2$. Any point on this circle is $(9 + 10\cos\alpha, 4 + 10\sin\alpha)$.
Now, $5x + 12y = 45 + 48 + 10(5\cos\alpha + 12\sin\alpha)$

$$= 93 + 130\left(\frac{5}{13}\cos\alpha + \frac{12}{13}\sin\alpha\right)$$

$$= 93 + 130\sin(\alpha+\beta)$$

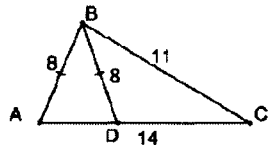
where $\sin\beta = \frac{5}{13}, \cos\beta = \frac{12}{13}$

Hence maximum value is 223.

Answer: B

6. In ΔABC, AB = 8, BC = 11, AC = 14, and D is a point on AC such that BD = 8. Then, if AD/DC equals $\frac{p}{q}$, where p, q are relatively prime integers then p + q equals

A. 196 B. 197 C. 199 D 201



Solution:

$$\cos A = \frac{14^2 + 8^2 - 11^2}{2\times 14\times 8} = \frac{139}{224} \qquad AD = 2\times 8\times\cos A = \frac{139}{14}.$$ Hence

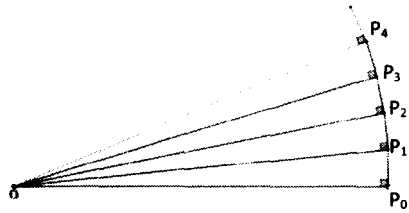
$$DC = 14 - \frac{139}{14} = \frac{57}{14}.$$

Therefore, $\frac{AD}{DC} = \frac{139}{57}$ giving p + q = 196.

Answer: A

7. The diagram shows a sequence of points P_0; P_1; P_2; P_3; P_4; ---; P_n;---- which spirals out around the point O. For any point P in the sequence, the line segment joining P to the next point is perpendicular to OP and has a length of 3 unit, as shown. Also, OP_0 = 29 unit. The smallest positive integer n for which OP_n is an integer measure in length, is

A. 35 B. 34 C. 38 D. 67



Solution:

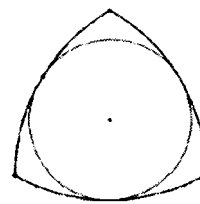
We have $OP_n^2 = OP_o^2 + n \times 3^2 = 29^2 + 9n = 841 + 9n --(1)$

Let OP_n = m be the smallest positive integer satisfying (1). $m^2 - 29^2 = (m - 29) \times (m + 29) = 9n$. This implies $m > 29$ and $m^2 - 29^2$ is a multiple of 9. Both (m – 29) and (m + 29) both cannot be divisible by 3. Hence either (m – 29) or (m + 29) is divisible by 9. (If m = 38, then n = (38 – 29)×(38 + 29)/9 = 67). We can choose m = 34 so that so that 34 + 29 = 63 is divisible by 9. Then n = 5×7 = 35. The least value of n is 35.

Answer: A

8. The curvy shape ABC shown in the figure below is called a Rouleaux's triangle (after the French engineer Franz Rouleaux). Its perimeter consists of three equal arcs AB, BC, CA each with the same radius and centred at the opposite vertex. If the common radius is 3 cm, what is the area in cm^2 of the inscribed circle?

A. $6\pi(2-\sqrt{3})$ B. $\frac{9\pi}{4}$ C. $2\pi(3-\sqrt{3})$ D. $\frac{3\pi}{4}$



Solution:

It is an equilateral triangle. Since each arc is of 60^0, the side of the equilateral triangle is 3 cm. The centre of the in-circle is the centroid of the equilateral triangle. The distance of the centroid from the vertex of the triangle is $3\times\frac{\sqrt{3}}{2}\times\frac{2}{3}=\sqrt{3}$ cm. The radius of the inner circle is $3-\sqrt{3}$.

Area of the inner circle is $\pi\times\left(3-\sqrt{3}\right)^2=6\pi\left(2-\sqrt{3}\right)\text{cm}^2$.

Answer: A

9. Let $x^2 - 4xy + 129y^2 = 2025$. Then the number of integer solutions (x, y) is

A. 2 B. 4 C. 6 D. none of these

Solution:

Consider the equation $x^2 - 4xy + 129y^2 - 2025 = 0$ to be a quadratic in x. To have integer solutions it's discriminant must be a perfect square. The discriminant is $D = 16y^2 - 4(129y^2 - 2025) = 8100 - 500y^2 = 500(16 - y^2) + 100$.
For $y = 0$, $D = 8100 = (\pm 90)^2$, and $x = \pm 45$.
For $y = \pm 1, \pm 2$, D is not a perfect square.
For $y = \pm 3$, $D = 3600 = (\pm 60)^2$. Again we have two solutions for x; $x = (\pm 12 \pm 60)/2$.
For $y = \pm 4$, $D = 100 = (\pm 10)^2$. Again we have two solutions for x; $x = (\pm 16 \pm 10)/2$
Totally **TEN solutions.**

Answer: D

10. A positive integer is said to be sorted if it has at least two digits and the digits altogether occur in either strictly ascending order or strictly descending order. Note: 20, 18, 349, 65 and 8421 are sorted whereas 287, 344, 10648, 2341

and 527 are not sorted. The total number of sorted positive integers is

A. 1004 B. 1515 C. 1024 D. 2026

Solution:

Number of numbers in ascending order is

$\sum_{k=2}^{9}\binom{9}{k}=36+84+126+126+84+36+9+1=502$.

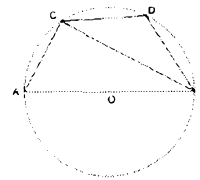
Note that in this the leading digit cannot be 0. Number of numbers in descending order is

$\sum_{k=2}^{10}\binom{10}{k}=45+120+210+252+210+120+45+10+1=1013$. Total number of such numbers is 502+1013=1515.

Answer: B

11. The diameter AB of the circle is 5 cm and BD = 4 cm. Also $\angle ABC = \angle CBD$. The area of the quadrilateral ABDC in cm^2 is

A. 10 B. 12 C. 8 D. 6.75



Solution:

In right ΔADB, $\angle ADB = 90$ and hence AD=3.

If $\angle ABC = \angle CBD = \alpha$ then $\cos 2\alpha = \frac{4}{5}$

In right ΔABC, $\sin\alpha = \frac{AC}{5}$

$\therefore AC = 5\sin\alpha$

$$= 5\sqrt{\frac{1-\cos 2\alpha}{2}}$$

$$= \frac{\sqrt{10}}{2}$$

Since $\angle ABC = \angle CBD = \alpha$, $AC=CD= \frac{\sqrt{10}}{2}$

Area of ABCD = (ABD) + (ACD)

$$= \frac{1}{2}.BD.AD + \frac{1}{2}.AC.CD.\sin(180-2\alpha)$$

$$= \frac{1}{2}.4.3 + \frac{1}{2}.\frac{\sqrt{10}}{2} + \frac{\sqrt{10}}{2}\sin 2\alpha$$

$$= 6 + \frac{5}{4} \times \frac{3}{5}$$

$=6.75\ cm^2$.

Answer: D

12. Let x be a real number chosen uniformly at random between 200 and 400. If $[\sqrt{x}]= 16$, then the probability that $[\sqrt{100x}] = 160$ is

A. 1/33 B. 1/11 C. 4/33 D. None of these.

Solution:

$[\sqrt{x}] = 16$ implies $16 \le \sqrt{x} < 17$, i.e. $256 \le x < 289$.
Hence, $25600 \le 100x < 28900$. But we need $[\sqrt{100x}] = 160$.
i.e., $160 \le \sqrt{100x} < 161$, $25600 \le 100x < 25921$. Under the condition that $\sqrt{x} = 16$, we need $[\sqrt{100x}] = 160$.
Required probability is $\frac{321}{3300} = \frac{107}{1100}$.

Answer: D

13. Let A = {1, 2, 3, 4, 5}. If f:A→A, then the number of f such that
f^3: A → A is the identity function, where f^3 is $f \circ f \circ f$ is

A. 3 B. 9 C. 10 D. 21

Solution:

Take any three members of A, say 3,4,5. If f(3)=4, f(4)=5, f(5)=3 and f(1)=1, f(2)=2; then f^3 will be an identity function. We have one more mapping of this set i.e. f(1)=1, f(2)=2, f(3)=5, f(5)=4, f(4)=3 which is also an identity mapping.

Now we have $\binom{5}{3}=10$ such subsets of A, each giving rise to 2 identity mapping, totaling to 20. Moreover f(1)=1, f(2)=2, f(3)=3, f(4)=4, f(5)=5 is a trivial case.

$\therefore$ Total 21 is the required number.

Answer: D

14. A rectangle is cut by several lines parallel to its sides to obtain 1000 rectangular pieces. We count the number of all pieces that come from edges and corners. Which of the following could NOT possibly be the number of corner and edge pieces?

A. 126 B. 136 C. 216 D. 316

Solution:

If a rectangle is cut into m×n smaller rectangular pieces by lines parallel to its sides, then the number of all pieces coming from the edges and corners is 2m + 2n – 4. 1000 can be written as 2×500, 4×250, 8×125, 10×100, 20×50, 40×25. The number of pieces coming from the edges and corners in each of these cases is 1000, 504, 262, 216, 136, 126. The only number in the four choices that does not occur is 316.

Answer: D

15. Let A be a subset of {1, 2, 3,..., 2020} such that if x is in A, 20x cannot be in A. What is the maximum cardinality of A?

A. 101 B. 1919 C. 1920 D. 100

Solution:

A = set of all n from 1, 2, ..., 2020 so that it is not a multiple of 20. Number of multiples of 20 is 101. Maximum cardinality of A is 1919.

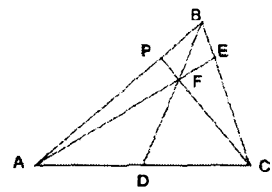
Answer: 1919

16. In Δ ABC, D is the midpoint of AC and E is a point on BC such that $\frac{BC}{BE} = 4$. Let BD and AE meet in F. If area of Δ ABF = 1sq. unit, then area of Δ ABC in square units is

A. 3 B. 4 C. 5 D. 6

Solution:

Join CP and produce it to cut AB in P.



[ABD] = [BCD], [AFD] = [CFD] and hence [AFB] = [CFB] = 1. [ABE] = (1/4)[ABC]. BY Ceva's theorem, $\frac{AD}{DC} \times \frac{CE}{EB} \times \frac{BP}{PA} = 1$. Therefore, $\frac{BP}{PA} = \frac{1}{3}$. Hence [BFP] = (1/4)[AFB] = ¼. So, [CBP] = [CFB] +[BFP] = 5/4.

[ABC] = 4×[CBP] = 5 square units.

Answer: C

17. Given that x, y are positive real numbers satisfying $2x^3+16y^3+\frac{1}{32}=3xy$. Then $\frac{1}{y}$ is -----.

A. 1/8 B. 8 C. 1/4 D. 4

Solution:

$x^3+8y^3+\frac{1}{64}=\frac{3}{2}xy$. Therefore $x^3+(2y)^3+\left(\frac{1}{4}\right)^3-3(x)(2y)\left(\frac{1}{4}\right)=0$.

Factorising the LHS we get

$\left(x+2y+\frac{1}{4}\right)\times\left(x^2+4y^2+\frac{1}{16}-2xy-\frac{y}{2}-\frac{x}{4}\right)=0$. As $x+2y+\frac{1}{4}>0$,

th second factor = 0, which leads to

$(2-2y)^2+\left(2y-\frac{1}{4}\right)^2+\left(x-\frac{1}{4}\right)^2=0$. This implies $y=\frac{1}{8}$ and $x=\frac{1}{4}$.

Answer: 8

18. The largest positive integer such that n^3 + 1340 is exactly divisible by n + 15 is -------.

A. 40 B. 160 C. 392 D. 2020

Solution:

n^3 + 3375 – 2035 should be divisible by n + 15. n^3 + 3375 = n^3 +15^3 is divisible by n + 15. Therefore, 2035 must be divisible by n + 15. Hence n + 15 must be either 5, 11, 37, 55, 185, 407, 2035. As we are looking for positive integer n, the first two are not possible. The largest n is 2020.

Answer: 2020

19. A 4 x 4 anti-magic square is an arrangement of the numbers 1 to 16 inclusive, in a square so that the totals of each of the four rows, four columns and two diagonals are ten consecutive numbers in some order. The diagram shows an incomplete anti-magic square. When it is completed what number will be at the place * ?

4	5	7	14
6	13	3	*
11	12	9	
10			

A. 16 B. 15 C. 8 D. 2

Solution:

First column sum is 31, first row sum 30, diagonal sum 39. Now 30 to 39 there are 10 consecutive numbers. So, the remaining numbers must come as row, column, and diagonal sum. Third row partial sum is 32. Second column partial sum is 30. The remaining five numbers to be used is 1, 2, 8, 15, 16. The arrangement is shown above. Number to be filled in * is 15.

Answer: 15

20. Consider a convex polygon with six sides in which no two diagonals are parallel and no three diagonals are concurrent. The number of points of intersection of the diagonals that lie outside the polygon is ---------.

A. 6 B. 5 C. 4 D. 3

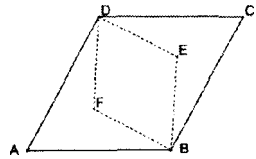
Solution: If ABCDEF is the hexagon, then AC and DF are diagonals which will meet outside. Similarly BD; EA and

CE and FB also meet outside the hexagon. Only three such points

Answer: 3

21. Rhombus ABCD is similar to quadrilateral BFDE. If the area of rhombus ABCD is 24 sq.units and $\angle ABC = 120^0$, then the area of quadrilateral BFDE is

A. 6 sq.units B. 8 sq.units
C. $8\sqrt{3}$ sq.units D. 12 sq.units



Solution:

AC x BD = 48 sq. units. $\angle DEB = 120^0$, $\angle EDF = 60^0$, $\angle EDC = 30^0$, $\angle DCE = 30^0$. Hence DE = EF = FD = EC = AF. Hence EF = AC/3. Hence area of DEBF = (BD × EF)/2 = BD × AC/6 = 48/6 = 8 sq. units.

Answer: B

22. If $f(x)\dfrac{1}{\sqrt[3]{x^2+2x+1}-2\sqrt[3]{x^2-1}+\sqrt[3]{x^2-2x+1}}$ then $\left\{\dfrac{1}{\sqrt{f(1)}}+\dfrac{1}{\sqrt{f(3)}}+\ldots+\dfrac{1}{\sqrt{f(2019)}}\right\}^3 = \ldots$

A. $\sqrt{2020}$ B. $\sqrt{2021}$ C. 2020 D. 2021

Solution:

$$\frac{1}{f(x)} = \sqrt[3]{x^2+2x+1}-2\sqrt[3]{x^2-1}+\sqrt[3]{x^2-2x+1}$$

$$= \left(\sqrt[3]{x+1} - \sqrt[3]{x-1}\right)^2$$

$$\frac{1}{\sqrt{f(x)}} = \sqrt[3]{x+1} - \sqrt[3]{x-1}$$

Now,

$$\frac{1}{\sqrt{f(1)}} = \sqrt[3]{2}\,,\ \frac{1}{\sqrt{f(3)}} = \sqrt[3]{4} - \sqrt[3]{2}, \frac{1}{\sqrt{f(5)}} = \sqrt[3]{6} - \sqrt[3]{4}....$$

$$...\frac{1}{\sqrt{f(2019)}} = \sqrt[3]{2020} - \sqrt[3]{2018}$$

Hence,

$$\left\{\frac{1}{\sqrt{f(1)}} + \frac{1}{\sqrt{f(3)}} + ... + \frac{1}{\sqrt{f(2019)}}\right\}^3$$

$$= \left\{\sqrt[3]{2} + \sqrt[3]{4} - \sqrt[3]{2} + \sqrt[3]{6} - \sqrt[3]{4} + ... + \sqrt[3]{2020} - \sqrt[3]{2018}\right\}^3$$

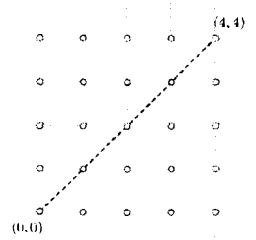
$$= \left\{\sqrt[3]{2020}\right\}^3$$

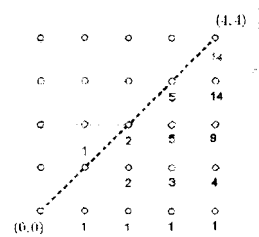
=2020

Answer: C

23. Find the number of shortest paths from (0, 0) to (4, 4) moving along the grid lines and not crossing the diagonal in the following grid, stays below the diagonal or at most touches it.

A. 13 B. 14 C. 15 D. 16





Solution:

Without crossing the diagonal, we have counted the number of ways of reaching the vertex with least number of steps. Finally, we get 14 at (4,4)

Answer: B

24. Let $X=\left\{x \mid -1 \leq \log_{\frac{1}{x}} 100 < \frac{-1}{3}, x \text{ integer}\right\}$. Then the number of nonempty subsets of X is --------.

A. 0 B. 1 C. 3 D. greater than 3

Solution:

X is an empty set. Hence the number of nonempty subsets is 0.

Answer: A

25. Each of the following sequences is an infinite Arithmetic Progression:

8, 15, 22, 29, 36, 43, · · ·
3, 14, 25, 36, 47, 58, · · ·
11, 24, 37, 50, 63, 76, · · ·

If M is the smallest positive integer present in all the three Arithmetic Progressions, then the sum of the digits of M is -----.

A. 17 B. 16 C. 15 D. 14

Solution:

If x is the least common element in the three APs, then, $x \equiv 8 \pmod 7 \equiv$
$1 \pmod 7 \equiv 3 \pmod{11} \equiv 11 \pmod{13}$.
We get $7k + 1 \equiv 3 \pmod{11}$, i.e., $7k \equiv 2 \pmod{11}$. Hence, $21k \equiv 6 \pmod{11}$. Therefore $k \equiv 5 \pmod{11}$. Let $k = 11b + 5$ for some $b \varepsilon \mathbf{W}$.
$7k + 1 = 77b + 36 \equiv 11 \pmod{13}$; $77b \equiv -25 \pmod{13} \equiv 1 \pmod{13}$. Hence $b = 12 \pmod{13}$. i.e., $b = 13r + 12$ and $k = 143r + 132 + 5 = 143r + 137$. Therefore, the common element $7k + 1 = 1001r + 959 + 1 = 1001r + 960$. Smallest value common is 960.

Answer: C

26. Ria is placing coloured beads on a string. She begins with one red, one white and one blue bead in that order. She continues with 1 red, 2 white and 3 blue beads. Keeping the same colour order, she places 1 red, 3 white and 6 blue beads on the string. In general, she follows the pattern that each set of red, white, and blue beads will consist of 1 red bead, n white beads and $n(n+1)/2$ blue beads. When she places the 2020th blue bead on the string what is the total number of white beads on the string? --------

A. 251 B. 252 C. 253 D. 255

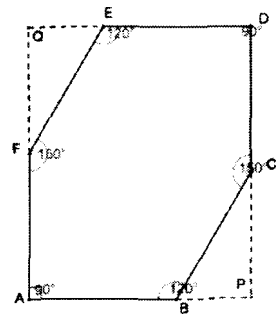
Solution:

Let the 2020^{th} blue bead be placed in nth stage. Then $1 + 3 + 6 + \ldots + n(n+1)/2 \geq 2020$. i.e., $n(n + 1)(n + 2)/6 \geq 2020$.
Since $n(n + 1)(n + 2) \geq 12120$. Since $21 \times 22 \times 23 \leq 12120 \leq 22 \times 23 \times 24 = 12144$, we get $n = 22$.
Number of white beads $= (1 + 2 + 3 + \ldots. + 22) = 253$.

Answer: C

27. A hexagon has consecutive angle measures of 90^0, 120^0, 150^0, 90^0, 120^0, and 150^0. If all of its sides are 4 units in length, and the area of the hexagon can be written as $m+n\sqrt{p}$, where m, n, p are positive integers and p is not a perfect square, the value of m + n + p is ---------.

A. 35 B. 27 C. 32 D. 37



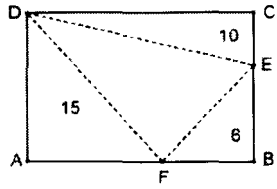
Solution:

AB = BC = CD = DE = EF = FA = 4 units. Area of hexagon = area of rectangle APDQ – 2 area of Δ BPC. Δ BPC is a right triangle with $30^0 - 60^0 - 90^0$ angles. Hence BP = 2 units and CP = $2\sqrt{3}$ units. [APDQ] = 6 x $(4 + 2\sqrt{3})$ = $24 + 12\sqrt{3}$. 2[BPC] = $4\sqrt{3}$. Required area = $24 + 8\sqrt{3}$. Hence m + n + p = 35.

Answer: A

28. Given the following figure of a rectangle ABCD with points E on BC and F on AB such that the area of ΔCED = 10 cm^2, area of ΔBEF = 6 cm^2 and area of ΔAFD = 15 cm^2, the area of ΔDEF in cm^2 is --------.

A. 31 B. 50 C. 19 D. None of these



Solution:

Let AB = a and BC = b. Let CE = x and AF = y. Since ax = 20, by = 30

We get $x=\frac{20}{a}$ and $y=\frac{30}{b}$. Then $\left(a-\frac{30}{b}\right)\left(b-\frac{20}{a}\right)=2[EBF]=12$. We get the equation $ab+\frac{600}{ab}-50=12$. i.e., $(ab)^2-62ab+600=0$.

Solving this quadratic in ab, we get $ab=\frac{62\pm\sqrt{62^2-2400}}{2}=\frac{62\pm\sqrt{1444}}{2}=(62+38/2=50)$ (since ab > 10 + 15 + 6 = 31). Hence [DEF] = 50 – 31 = 19cm^2.

Answer: C

29. The roots of the equation $a^2-pa+q=0$ are $\sqrt[3]{81-33\sqrt{6}}$ and $\sqrt[3]{81+33\sqrt{6}}$. Then pq is ------.

A. 6 B. 18 C. 3 D. 9

Solution:

$p=\sqrt[3]{81-33\sqrt{6}}+\sqrt[3]{81+33\sqrt{6}}$ and

$q=\sqrt[3]{81-33\sqrt{6}}\times\sqrt[3]{81+33\sqrt{6}}=\sqrt[3]{27}=3$

$p=\sqrt[3]{\left(3-\sqrt{6}\right)^3}+\sqrt[3]{\left(3+\sqrt{6}\right)^3}=6$

Hence pq = 18

Answer: B

30. In a right triangle, c, h are the lengths of hypotenuse and the altitude on the hypotenuse, respectively. If r is the radius of the in-circle of the triangle, then which of the following relation holds true?

A. $c = \frac{2r^2}{(h-2r)}$ B. $c = \frac{r+r^2}{(h-2r)}$ C. $c = \frac{5r^2}{(5h-11r)}$ D. $c = \frac{22r^2}{(h+2r)}$

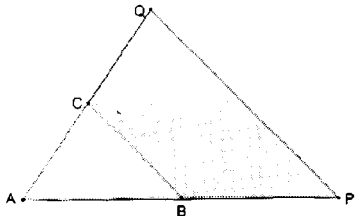
Solution:

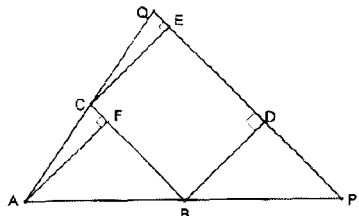
Let a, b be the legs of the right triangle. Then, $a^2 + b^2 = c^2$, $ab = ch$ and
$a+b-c = 2r$. $(a+b)^2 = c^2+2ch = (c+2r)^2$. Hence, we get $2r^2 = c(h-2r)$.
So, $c = \frac{2r^2}{(h-2r)}$.

Answer: A

FINAL - GAUSS CONTEST
NMTC AT PRIMARY LEVEL
V & VI STANDARDS

1. In Δ ABC, the side AB is extended to P and AC is extended to Q such that AB = BP and AC = CQ, as shown. If the area of the quadrilateral is 21 cm2, then find the area of Δ ABC.





Solution:

Since AB = BP and AC = CQ, BC ∥ PQ, (By converse of basic proportionality theorem). Drop BD and CE perpendicular to PQ and AF perpendicular to BC. Since BC ∥ PQ, AF ∥ CE and AF ∥ BD. Hence ΔCAF ≡ ΔQCE and ΔBAF ≡ ΔPBD. Hence [ABC] = [CEQ] + [BDP] and area of rectangle BCED = BC×BD = 2[ABC]. [PBCQ] = 3 [ABC] = 21 cm^2. Hence, [ABC] = 7 cm^2.

2. Arrange the following fractions in ascending order : $\frac{10}{31}, \frac{25}{61}, \frac{17}{71}, \frac{37}{73}$. Justify with reasons.

Solution:

It is easy to compare the reciprocals of the given fractions. Thereafter, we can compare the given fractions.

Consider $\frac{31}{10} = 3\frac{1}{10}$. This value lies between 3 and 4.

Now, $\frac{61}{25} = 2\frac{11}{25}$. This value lies between 2 and 3.

$\frac{71}{17} = 4\frac{3}{17}$. This value lies between 4 and 5.

$\frac{73}{37} = 1\frac{36}{37}$. This value lies between 1 and 2.

$\therefore$ We have $\frac{71}{17} > \frac{31}{10} > \frac{61}{25} > \frac{73}{37}$. Therefore, $\frac{17}{71} < \frac{10}{31} < \frac{25}{61} < \frac{37}{73}$.

Alternate Solution:

We have $\frac{17}{71} = \frac{51}{213} < \frac{60}{213} < \frac{60}{186} = \frac{10}{31} \Rightarrow \frac{17}{71} < \frac{10}{31}$.

Also, we have $\frac{10}{31} = \frac{50}{155} < \frac{50}{122} = \frac{25}{61} \Rightarrow \frac{10}{31} < \frac{25}{61}$.

Also, we have $\frac{25}{61} = \frac{100}{244} < \frac{111}{244} < \frac{111}{219} = \frac{37}{73} \Rightarrow \frac{25}{61} < \frac{37}{73}$.

Therefore, $\frac{17}{71} < \frac{10}{31} < \frac{25}{61} < \frac{37}{73}$.

Note that we have used the following facts while comparing fractions.

If the numerators of two fractions are the same, then the fraction with smaller denominator is greater.

If the denominators of two fractions are the same, then the fraction with greater numerator is greater.

3. Let M be the sum of all possible 4 digit numbers with different digits formed from digits 1, 2, 3, 4. Let N be the sum of all possible 4 digit numbers with different digits formed from digits 2, 4, 6, 8. Find the value of $\frac{N}{M}$.

Solution:

For every possible 4 digit number with different digits form 1,2,3,4, we have double of it which is a 4 digit number with different digits form 2,4,6,8. Few examples are $1234 \times 2 = 2468$; $2413 \times 2 = 4826$; $3124 \times 2 = 6248$; etc...

$\therefore M \times 2 = N \Rightarrow \frac{N}{M} = 2.$

4. Find two hundred digit numbers with *non-zero* digits and sum of the digits 125, that is divisible by the sum of its digits. Explain the process by which you arrived the answer.

Solution:

By property of divisibility by 125, any positive integer having the last 3 digits forming a number which is divisible by 125, is also divisible by 125. (no matter what the other leading digits, if any, are) Let us construct a 100 digit number with non-zero digits that is divisible by 125.

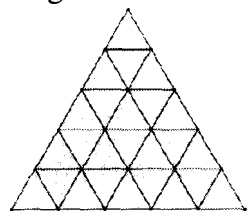
Consider the 100-digit multiple of 125: 111111 ····· 111125, which has 1 repeated 97 times followed by 125. The sum of the digits is $97 \times 1 + 1 + 2 + 5 = 105$. We can fine tune the number to have the sum of the digits to be 125 by increasing its first 4 digits each by 5 so that we get 6666 11 ····· 111125, having 4 sixes followed by 93 ones followed by 125.

Now, the sum of the digits is $6 + 6 + 6 + 6 + 93 \times 1 + 1 + 2 + 5 = 24 + 93 + 8 = 125$.

Thus, we have constructed one such 100 digit number with non-zero digits that is divisible by the sum of its digits.

Note that we can change the leading digits any which way we want so that the sum of the digits is 125. We will have infinitely many solutions.

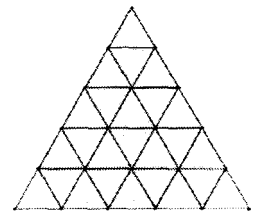
5. The diagram shows a grid of twenty five identical equilateral triangles. How many different rhombuses can be formed from two adjacent small triangles?



Solution:

Let us look at the figure:

Row 1 →
Row 2 →
Row 3 →
Row 4 →
Row 5 →



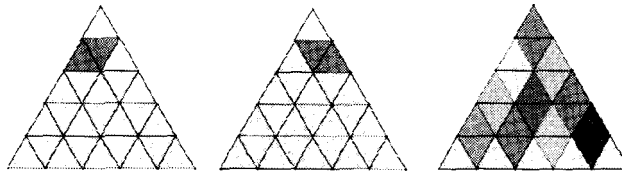
Note that rows numbered 1, 2, 3, 4, 5 have 1, 3, 5, 7, 9 equilateral triangles, respectively. In the figure, we need to find the number of two adjacent equilateral triangles together making a rhombus.

We can classify into cases for collecting rhombuses, as follows:

- Case 1: Two adjacent equilateral triangles forming a rhombus within a row
- Case 2: Two adjacent equilateral triangles forming a rhombus between two adjacent rows
- First case: Two adjacent equilateral triangles forming a rhombus within a row

There will be collection of 0, 2, 4, 6, 8 such rhombuses in 1st, 2nd, 3rd, 4th and 5th rows respectively.

There will be a collection of 2 such rhombuses in the 2nd row, as shown. Similarly, 4, 6, 8 from the remaining rows, a total of 20 rhombuses.



- Second case: This is simply looking at adjacent equilateral triangles in consecutive rows. See the third figure above. There are 10 such rhombuses.

We get a total of 30 rhombuses.

6. A sequence has all 4-digit numbers from 1000 till 2021 that contains 3 even digits and 1 odd digit. It has numbers in ascending order with no number repeating. The sequence is : 1000, 1002, 1004, 1006, ..., 2018, 2021. How many numbers are there in the sequence?

Solution:

In 1000's, that is from 1000 to 1999, all the positive integers start with the odd digit 1. We need all the remaining three digits in this lot to be even for the numbers to be a part of the sequence. Each digit in the hundred's place, ten's place and the unit's place has five possibilities between the digits 0, 2, 4, 6 and 8.

There are $5 \times 5 \times 5 = 125$ numbers in 1000's as a part of the finite sequence.

In 2000's, the numbers of the sequence are: 2001, 2003, 2005, 2007, 2009, 2010, 2012, 2014, 2016, 2018, 2021.

There are 11 numbers in 2000's as a part of this sequence; it finally ends with 2021.

Total number of numbers in the sequence is 125 + 11 = 136.

7. It is observed that Wednesday appears across dates 3, 5, 6 only once in a particular calendar year. Also, Wednesday appears exactly two times across each date 1, 2, 7 in that year. If the year is not a leap year, what is the day on January 1st of that year?

Solution:

A Wednesday falls on exactly one among the dates of 1, 2, 3, 4, 5, 6, 7 of each month of any calendar year. Let us tabulate a calendar for a normal year if Wednesday falls on the first of January for that year:

Normal Year	*Jan*	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*Jul*	*Aug*	*Sep*	*Oct*	*Nov*	*Dec*
Wednesday	*1*	*5*	*5*	*2*	*7*	*4*	*2*	*6*	*3*	*1*	*5*	*3*

But given that Wednesday appears across dates 3, 5, 6 only once in a particular calendar year and it appears exactly two times across each date 1, 2, 7 in that year and therefore appears exactly three times across date 4.

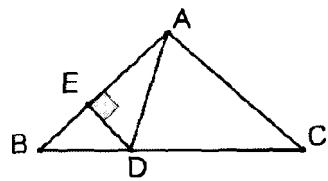
From the table, we observe that January 1st of that particular normal year cannot be a Wednesday. Let us extend the table for other 6 possible cases of Wednesday and analyse the same, as follows:

Normal Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Wednesday	1	5	5	2	7	4	2	6	3	c	5	3
Wednesday	2	6	6	3	1←8	5	3	7	4	2	6	4
Wednesday	3	7	7	4	2	6	4	1←8	5	3	7	5
Wednesday	4	1←8	1←8	5	3	7	5	2	6	4	1←8	6
Wednesday	5	2	2	6	4	1←8	6	3	7	5	2	7
Wednesday	6	3	3	7	5	2	7	4	1←8	6	3	1←8
Wednesday	7	4	4	1←8	6	3	1←8	5	2	7	4	2

Note: You can just push every date number forward by 1 from every previous row to the next row, as indicated in the table.

Only the 7th row of dates (last row of the table) satisfies all the given conditions.
Hence, January 7th of that particular normal year was a Wednesday.
Therefore, January 1st and 8th of that year was a Thursday.

8. Δ ABC is an isosceles triangle with AB = AC. D is a point on BC such that AB = CD. Draw DE $\perp$ AB at E. Show that $2 \angle ADE = 3 \angle B$.

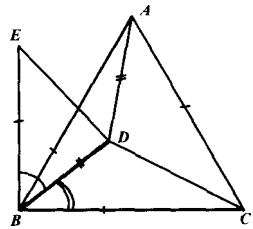


Solution:

Let $\angle B = \angle C = \theta$. Then $\angle A = 180 - 2\theta$. Since Δ CAD is isosceles, $\angle ADC = \angle DAC = (180 - \theta.)/2. = 90 - \theta./2$. $\angle BAD = \angle BAC - \angle DAC = 180 - 2\theta. - (90 - \theta./2) = 90 - 3\theta/2$. Hence, $\angle ADE = 90 - \angle BAD = 3\theta/2$. Hence, $2\angle ADE = 3\angle B$.

FINAL - KAPREKAR CONTEST
NMTC AT SUB JUNIOR LEVEL
VII & VIII STANDARDS

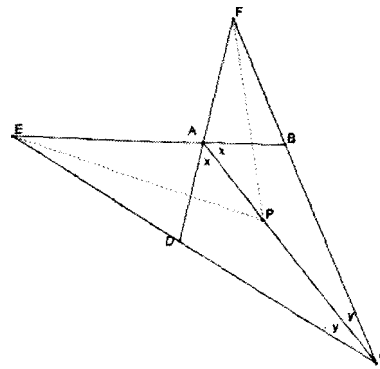
1. ΔABC is an equilateral triangle. D is a point inside ΔABC such that AD = BD. Choose E such that BE = AB and BD bisects $\angle CBE$. Find $\angle BED$. Diagram given below.



Solution:

In ΔCBD and ΔEBD, BC = AB = BE, BD common, $\angle CBD = \angle EBD$. By SAS, $\Delta CBD \equiv \Delta EBD$. Hence, $\angle DCB = \angle DEB$. Since ABC is equilateral and DA = DB, D lies on the perpendicular bisector of AB which is the altitude through C. CD is the altitude and the angle bisector of $\angle C$. Hence, $\angle DCB = 30^0 = \angle DEB$. Therefore $\angle BED = 30^0$.

2. Given a quadrilateral ABCD, the diagonal AC bisects both $\angle A$ and $\angle C$. If AB and DC extended intersect at E, and AD and BC extended intersect at F, show that for any point P on line AC, PE = PF.



Solution:

In Δ ABC and Δ ADC, AC is common, $\angle BAC = \angle DAC$, $\angle BCA = \angle DCA$. Hence by ASA, Δ ABC $\equiv \Delta$ ADC.
Hence AB = AD and BC = DC.
In Δ ABF and Δ ADE, AB = AD, $\angle BAF = \angle DAE$ and $\angle ABF = x + y = \angle ADE$. By ASA, Δ ABF $\equiv \Delta$ ADE.
Hence, BF = DE and AF = AE.
In Δ APF and Δ APE, AP common, AF = AE and $\angle PAF = \angle PAE$.
By SAS, Δ APF $\equiv \Delta$ APE. Hence, PE = PF.

3. Twenty-seven balls labelled from 1 to 27 are distributed in three bowls red, blue, and yellow. What are the possible values of the number of balls in the red bowl, if the average of the labels in the red, blue, and yellow bowl are 15, 3, and 18 respectively?

Solution:

Let m be the number of balls in red bowl and n the number of balls in blue bowl. Then the number of balls in yellow bowl is $27 - m - n$. $1 \leq m, n \leq 25$
$15m + 3n + 18(27 - m - n) = 27 \times 14$; $3m + 15n = 27 \times 4$; $m + 5n = 36$.

Therefore, (m, n) = (1, 7), (6, 6), (11, 5), (16, 4), (21, 3). Among these n = 7 and 6 will not give an average of 3 in blue bowl (The least labels are 1, 2, ..., 7 giving an average of 4). Hence the possible number of balls in red bowl is 11, 16 and 21.

4. Prove that the integer $53\times83\times109 + 40\times66\times96$ is not a prime number.

Solution:

$$53\times83\times109 + 40\times66\times96 = 53(96^2 - 13^2) + (53^2 - 13^2)96$$
$$= 53^2.\ 96 + 96^2.\ 53 - 13^2(96 + 53)$$
$$= 53\ .\ 96\ (96 + 53) - 13^2\ .\ 149$$
$$= 53.\ 96\ .\ 149 - 169.\ 149$$
$$= 149(53.\ 96 - 169).$$

149 a factor of the given number. Hence not prime.

5. Let a, b, c, and d be real numbers such that $a^3 + b^3 + c^3 + d^3 = a + b + c + d = 0$. Prove that sum of a pair of these numbers is equal to 0.

Solution:

If $a + b = 0$, we are done. Assume $a + b \neq 0$. Then $c + d \neq 0$.

$a^3 + b^3 = -(c^3 + d^3)$ and $a + b = -(c + d)$.

Dividing the first by the second we get,

$$a^2 + b^2 - ab = c^2 + d^2 - cd$$
$$(a + b)^2 - 3ab = (c + d)^2 - 3cd$$
$$\Rightarrow ab = cd$$
$$\Rightarrow (a + b)^2 - 4ab = (c + d)^2 - 4cd$$
$$\Rightarrow (a - b)^2 = (c - d)^2$$
$$\Rightarrow (a - b) = (c - d) \text{ or } (a - b) = -(c - d).$$

If we use $a - b = c - d$,
we get $a + d = b + c = 0$
since $a + b + c + d = 0$.
If we use $a - b = -c + d$,
we get $a + c = b + d = 0$,
since $a + b + c + d = 0$.

6. Find any hundred digit number with *non-zero* digits, that is divisible by the sum of its digits. Explain the process by which you arrived the answer.

Solution:

By property of divisibility by 125, any positive integer having the last 3 digits forming a number which is divisible by 125, is also divisible by 125. (no matter what the other leading digits, if any, are)

Let us construct a 100 digit number with non-zero digits that is divisible by 125.

Consider the 100-digit multiple of 125: 111111 · · · · · · 111125, which has 1 repeated 97 times followed by 125. The sum of the digits is $97 \times 1 + 1 + 2 + 5 = 105$. We can fine tune the number to have the sum of the digits to be 125 by increasing its first 4 digits each by 5 so that we get 6666 11 · · · · · · 111125, having 4 sixes followed by 93 ones followed by 125.

Now, the sum of the digits is $6 + 6 + 6 + 6 + 93 \times 1 + 1 + 2 + 5 = 24 + 93 + 8 = 125$.

Thus, we have constructed one such 100 digit number with non-zero digits that is divisible by the sum of its digits.

Note that we can change the leading digits any which way we want so that the sum of the digits is 125. We will have infinitely many solutions.

Alternate Solution:

By property of divisibility by 128, any positive integer having the number formed by the right most block of 7-digits divisible by 128, is also divisible by 128.

Let us construct a 100 digit number that is divisible by 128.

Consider the 100-digit number 111111111 ····· 1111024256, with 93 ones, followed by 1024256 which is divisible by 128.

Sum of the digits = 93 + 1 + 0 + 2 + 4 + 2 + 5 + 6 = 113. Increase the leading digits to get an additional 15 in the sum. Change the two leading digits to 9 and 8.

98111...1111024256 which has sum of the digits 9 + 8 + 91+20 = 128. Again, we get infinitely many solutions.

Alternate Solution:

Look at M = 11111111...1110 which has 33 blocks of 111 and a trailing 0. Now, M is divisible by 111. But sum of the digits of M is 99! But we want non-zero digits. Let us look at the tail. Add 444 to M we get N = 111111...111554, which has 32 blocks of 111, followed by1554 which equals M + 444 which is divisible by 111 = the sum of the digits of N.

7. $\frac{a}{b}$ is an improper fraction in its simplest form. If its equivalent mixed fraction is $d\frac{c}{b}$, such that d + c = 9 and a + b = 100, then find all such fractions.

Solution:

a + b = 100 and $\frac{a}{b} = \frac{bd+c}{b}$. Hence a = bd + c.

$\therefore$ 100 – b = bd + c

$\Rightarrow$ b (d+1) + c = 100

$\Rightarrow$ b (10–c) + c = 100

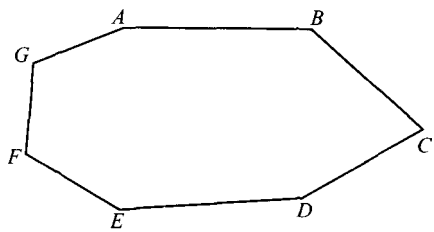
Now,

c	b	d	a	$\frac{a}{b}$	
1	11	8	89	$\frac{89}{11}$	
2	Fraction				Invalid
3	Fraction				Invalid
4	16	5	84	$\frac{84}{16}$	Invalid, not a reduced fraction
5	19	4	81	$\frac{81}{19}$	
6	Fraction				Invalid
7	31	2	69	$\frac{69}{31}$	
8	46	54			Invalid, not a reduced fraction

Only three valid solutions: $\frac{89}{11}, \frac{81}{19}, \frac{69}{31}$.

8. ABCDEFG is a regular heptagon. How many quadrilaterals can be obtained from the heptagon satisfying the following properties:
   - All the four vertices of the quadrilateral are some vertices of the heptagon
   - Two sides of the quadrilateral are also sides of the heptagon, but not the other two.

Solution:



ABCDEFG is the heptagon.

Case 1: Two adjacent sides of the heptagon are used.
ABC, BCD, CDE, DEF, EFG, FGA, GAB are the possible cases. But we should not
have any other side of the heptagon as the side of the quadrilateral. Hence if we

consider ABC, then D must not be chosen, nor should G. So, the possibilities are
ABCE, or ABCF. So, we have two quadrilaterals with ABC. Similarly for the other
six also. Totally we have 7 x 2 = 14 such quadrilaterals.

Case 2. The two sides of the heptagon which are also sides of the quadrilateral are not
adjacent.
If side AB is chosen, then BC and GA are out of consideration. We cannot use CD as
it will result in the quadrilateral ABCD. Similarly, FG cannot be used as it will lead to
the quadrilateral BAGF. So only possibilities are DE or EF as the other side. Hence,
we get the quadrilaterals ABDE or ABEF. Here again, we get 7 x 2 = 14
quadrilaterals. But we have overcounting here! ABDE will be counted in AB as well
as DE. Hence, we have only 14/2 = 7 such quadrilaterals.
Total is 21 such quadrilaterals

FINAL - BHASKARA CONTEST
NMTC AT JUNIOR LEVEL
IX & X STANDARDS

1. The first five terms of a sequence are 1, 2, 3, 4 and 5. From the sixth term on, each term is 1 less than the product of all the preceding ones. Prove that the product of the first 70 terms is equal to the sum of their squares.

Solution:

Let $a_k=k$, for k = 1, 2, 3, 4, 5.
By the given condition,
$a_{k+1}=a_k a_{k-1}\ldots a_2 a_1-1$, for k = 5, 6, ..., 69.
Since $a_{k+1}+1= a_k a_{k-1}\ldots a_2 a_1$, we get $a_{k+1}a_k a_{k-1}\ldots a_2 a_1=a_{k+1}(a_{k+1}+1)={a_{k+1}}^2+a_{k+1}$.

Putting k = 69 we get,

$$\Pi_{i=1}^{70} a_i = 1 = {a_{70}}^2+a_{70}$$
$$= {a_{70}}^2+ a_{69}a_{68}\ldots a_2 a_1-1$$
$$= {a_{70}}^2+ a_{69}(a_{69}+1)-1$$
$$= {a_{70}}^2+ {a_{69}}^2+ a_{69}-1 = {a_{70}}^2+ {a_{69}}^2+a_{68}\ldots a_2 a_1-1-1$$
$$= {a_{70}}^2+ {a_{69}}^2+a_{68}(a_{68}+1)-2$$
$$= {a_{70}}^2+ {a_{69}}^2+{a_{68}}^2+ a_{68}-2$$
$$= \ldots\ldots$$
$$= {a_{70}}^2+ {a_{69}}^2+{a_{68}}^2\ldots+ {a_6}^2+a_6-64$$
$$= {a_{70}}^2+ {a_{69}}^2+{a_{68}}^2\ldots+ {a_6}^2+ 1.2.3.4.5-1-64$$
$$= {a_{70}}^2+ {a_{69}}^2+{a_{68}}^2\ldots+ {a_6}^2+ 5^2+4^2+3^2+2^2+1^2.$$

2. The number N equals the product of 100 different positive integers. Show that N has at least4951 different divisors (including 1 and the number itself). What kind of numbers N will have exactly 4951 divisors, if such numbers exist?

Solution:

We show by induction that if N is the product of k distinct integers, then N has at least $\frac{k(k-1)}{2}+1$ distinct divisors. The base case N = 1 is trivial. Suppose that $N = a_1 \times a_2 \times \ldots \times a_{k+1}$ is the product of k + 1 distinct integers. We can assume that a_{k+1} is the largest of the factors. Let $M = a_1 \times a_2 \times \ldots \times a_k$. By induction hypothesis, M has at least $\frac{k(k-1)}{2}$ distinct divisors. Note that $\frac{M}{a_i} \times a_{k+1}$ is a divisor of N and all these divisors are larger than M (since $a_{k+1} > a_i$). Thus, N has at least $\frac{k(k-1)}{2}+1+k=\frac{k(k+1)}{2}+1$ divisors and this completes the induction.

When N is a product of 100 distinct integers, then N has at least $\frac{100\times 99}{2}+1=4951$ distinct divisors.

If p is any prime number, then for $N = 1 \times p \times p^2 \times p^3 \times \ldots \times p^{99} = p^{4950}$ has 4951 divisors exactly.

3. Find all positive integers k such that the product of the decimal digits of k equals $\frac{25}{8}k-211$.

Solution:

Let P(k) be the product of all the decimal digits of positive integer k.

Given that $P(k) = \frac{25}{8}k-211$. By property $0 \le P(k) \le k$.

$$\therefore 0 \le \frac{25}{8}k-211 \le k \Rightarrow 211 \le \frac{25}{8}k \le k + 211$$

$\Rightarrow 1688 \leq 25k \leq 8k + 1688$

$\Rightarrow k \geq \frac{1688}{25} = 67.52$ and $17k \leq 1688$

$\Rightarrow k \geq 68$ and $k \leq 99$, k being an integer.

$\therefore$ k is a two-digit number between 68 and 99, both inclusive.

Let A, B be the tens and units place digits of k.

We can now rewrite the given condition as:

$A \times B = \frac{25}{8}(10A + B) - 211$

$\Rightarrow$ 8 divides (10A + B) $\Rightarrow$ B is even and non-zero $\Rightarrow \frac{25}{8}$ $(10A+B)$ must be odd.

$\Rightarrow$ 10A + B is a multiple of 8 by an odd number.

Possible values of k = 10A + B are $8\times9 = 72$ and $8\times 11 = 88$.
When k = 72, P(k) = 14 and $258k-211 = 225 - 211 = 14$
When k = 88, P(k) = 64 and $258k-211 = 275 - 211 = 64$.

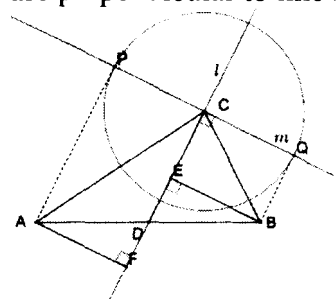
Only two values of k are 72 and 88

4. Given three non-collinear points A, B, C, construct a circle with centre C such that the tangents from A and B to the circle are parallel.

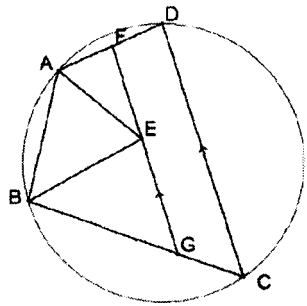
Solution:

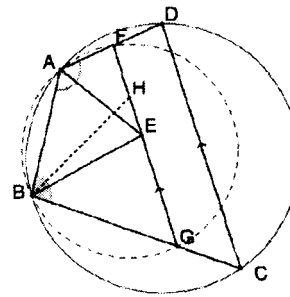
Let line l be the line AC where D is the mid-point of AB. Drop BE and AF perpendicular to l. In right Δ ADF and right Δ BDE, we have AD = BD and $\angle$FAD = $\angle$EBD. Hence, Δ ADF $\equiv \Delta$ BDE and so AF = BE. Let line m be perpendicular to line l through C. Drop AP and BQ perpendicular to m from A and B. As APCF is a rectangle AF = CP. As BQCE is a rectangle BE

= CQ. Hence CP = CQ. Draw a circle with C as centre and radius CP = CQ. For this circle AP and BQ are tangents and are parallel as both are perpendicular to line *m*.



5. Let ABCD be a cyclic quadrilateral where E is the point of intersection of the angle bisectors of ∠A and ∠B. Let a line through E parallel to CD meet AD, BC at F and G respectively. Prove that FG = AF + BG.





Solution:

Choose a point H on FG such that BG = GH. As we need to prove that FG = AF+BG = AF+GH, it is enough to prove that AF = FH. As FG ∥ DC, quadrilateral AFGB is cyclic. Since BG = GH, ∠BHG = ∠BHE = ∠HBG = ½∠FGC = ½∠BAD = ∠BAE. Hence A, B, E, H are con-cyclic. Hence, ∠FHA = ∠ABE = ½∠ABG = ½∠DFH = ½(180 - ∠AFH). Hence ∠FHA = ∠FAH.

Therefore AF = FH.

6. Find all positive integers $n < 2021$ whose square ends with 444.

Solution:

n is at the most a 4 digit number with digits a, b c, d. Note that since n^2 has the last 3 digits to be 444, d is either 2 or 8. The smallest number is known as the 2 digit number 38 ending in 8.

Case (i) $d = 2$.

Then $n = 1000a + 100b + 10c + 2$.

Hence $n^2 = (1000a + 100b + 10c + 2)^2$

$= 10^6a^2 + 10^4b^2 + 100c^2 + 4 + 2\times10^5ab + 2\times10^4ac + 4000a + 2\times10^3bc + 400b + 40c$.

The last 3 digits are given by only $100c^2 + 400b + 40c + 4$. Units place is 4. The tens place is given by 4c, as the other two terms have 0 in the tens place.

Therefore, $4c = 4 \Longrightarrow c = 1$ or 6.

If $c = 1$, then the hundreds place is given by the unit's place of $4b + 1$ which is odd and cannot be 4.

If $c = 6$, then the hundreds place is given by the unit's place of $36 + 4b + 2$ which is 4. Hence 4b must have 6 as its unit's place. i.e., $b = 4$ or 9. The leading digit can be 0 or 1 only as $n < 2021$. Hence the possible numbers are a462 or a962, where $a = 0, 1$.

We have 4 numbers: **462, 1462, 962, 1962**

Case (ii) $d = 8$.

Then $n = 1000a + 100b + 10c + 8$.

Hence $n^2 = (1000a + 100b + 10c + 8)^2$
$= 10^6a^2 + 10^4b^2 + 100c^2 + 64 + 2\times10^5ab + 2\times10^4ac + 16000a + 2\times10^3bc + 1600b + 160c.$

The last 3 digits are given by only $100c^2 + 1600b + 160c + 64$. Units place is 4. The tens place is given by $16c + 6$, as the other two terms have 0 in the tens place. Hence $16c + 6$ has unit's place 4. So, $16c$ has unit's place 8. So, c is 3 or 8.

If $c = 3$, then the hundreds place is given by the unit's place of $9 + 16b + 4 + 1$. But this is even only for $b = 0$ or 5. Hence the solutions are a038 or a538 for $a = 0, 1$.

Four solutions: **38, 1038, 538, 1538**

If $c = 8$, then the hundreds place is given by the unit's place of $64 + 16b + 12 + 1$ which is odd and hence no solution.

In all we have 8 solutions **38, 462, 538, 962, 1038, 1462, 1538, 1962.**

Alternate Solution:

Let x be the positive integer such that $x^2 = 1000y + 444$ for some $y \in N$.

Note: 444 is not a perfect square.

We have $x^2 = 1000(y - 1) + 1444 \Rightarrow x^2 - 38^2 = 2^3\times5^3 (y - 1)$ since, $38^2 = 1444$.

$\therefore (x - 38)(x + 38) = 2^3\times5^3 (y - 1)$. Hence, $\frac{(x-38)(x+38)}{2^3\times5^3} = y-1$.

For the numerator to be divisible by 2^3, x must be even. If $x \equiv 0 \pmod 4$, then the numerator is not divisible by 2^3. Therefore, $x \equiv 2 \pmod 4$ and hence both $(x - 38)$ and $(x + 38)$ are divisible by 4.

Since $(x - 38)(x + 38)$ is divisible by $5^3 = 125$, 125 divides either $(x - 38)$ or $(x + 38)$.

Thus either $(x - 38)$ or $(x + 38)$ is divisible by 500.

If $x - 38$ is divisible by 500, the possible values of x = **38, 538, 1038, 1538.**

If $x + 38$ is divisible by 500, the possible values of x = **462, 962, 1462, 1962.**

7. Prove, for any real number c, the equation $x(x^2 - 1)(x^2 - 10) = c$ cannot have five integer solutions.

Solution:

For $c = 0$, we have two irrational roots. Hence $c \neq 0$. Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ be the five integer roots of $x(x^2 - 1)(x^2 - 10) - c = x^5 - 11x^3 + 10x - c = 0$. Since the roots are integer, c must be an integer and non-zero.

We get the following relations:

$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 0$

$\Sigma_{1 \le i < j \le 5} \alpha_i \alpha_j = -11$

$\Sigma_{1 \le i < j < k \le 5} \alpha_i \alpha_j \alpha_k = 0 \Rightarrow \Sigma_{1 \le i < j \le 5} \frac{1}{\alpha_i \alpha_j} = 0$ (dividing by $\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 = c$)

Similarly, dividing the sum of the products of the roots taken four at a time = 10 by $\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 = c$, we get

$$\sum_{i=1}^{5} \frac{1}{a_i} = \frac{10}{c} \Rightarrow \sum_{i=1}^{5} \frac{1}{a_i^2} = \frac{100}{c^2}$$

Also, we get $\sum_{i=1}^{5} \alpha_i^2 = \left(\sum_{i=1}^{5} \alpha_i^2\right) - 2\sum_{1 \le i < j \le 5} \alpha_i \alpha_j = 22$

$\left(\sum_{i=1}^{5} \alpha_i^2\right)\left(\sum_{i=1}^{5} \frac{1}{\alpha_i^2}\right) \ge 25$ (By A.M – H.M inequality)

$\Rightarrow 22 \times \frac{100}{c^2} \ge 25 \Rightarrow c^2 \le 88.$

Hence, c = ±9, ±8, ±7, ±6, …., ±1. As the sum of the roots is 0, their product must be Even. Hence c = ±8, ±6, ±4, ±2

Note that we cannot have five integer numbers with sum 0 and whose product is c, for c = ±8, ±6, ±4, ±2, except c = ±6, For c=6, we have factors 1, 2, -3, -1, 1 but, these do not satisfy the equation. For c = -6, we have -1, -2, 3, 1, -1. Again, they do not satisfy the equation.

Hence, no such c exists.

8. The reals α and β are such that $\alpha^3 - 3\alpha^2 + 5\alpha = 1$ and $\beta^3 - 3\beta^2 + 5\beta = 5$ are satisfied. Find $\alpha + \beta$.

Solution:

$\alpha^3 - 3\alpha^2 + 5\alpha - 1 = 0$; Clearly $0 < \alpha < 1$.

The cubic expression is negative for $\alpha < 0$ and positive for $\alpha > 1$

$\beta^3 - 3\beta^2 + 5\beta - 5 = 0$; Clearly $1 < \beta < 2$.

Adding these two we get,

$(\alpha^3+\beta^3) - 3(\alpha^2+\beta^2) + 5(\alpha+\beta)-6 = 0.$

$(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta) - 3((\alpha+\beta)^2 - 2\alpha\beta) + 5(\alpha+\beta) - 6 = 0.$

$(\alpha+\beta)((\alpha+\beta)^2 - 3\alpha\beta) - 3((\alpha+\beta)^2 - 2\alpha\beta) + 5(\alpha+\beta) - 6 = 0.$

$(\alpha+\beta)^3 - 3(\alpha+\beta)^2 + 5(\alpha+\beta) - 6 + 3\alpha\beta(2-\alpha-\beta) = 0.$

Now $x^3 - 3x^2 + 5x - 6 = 0$ for $x = 2$.

Hence $(\alpha+\beta) = 2$ makes $(\alpha+\beta)3 - 3(\alpha+\beta)2 + 5(\alpha+\beta)-6=0$ and also $2\alpha\beta(2-\alpha-\beta) = 0.$

Hence, $\alpha+\beta = 2.$

FINAL - RAMANUJAN CONTEST
NMTC AT INTER LEVEL
XI & XII STANDARDS

1. Find a polynomial p(x) with the least possible degree which satisfies the conditions:
   (a) All the coefficients of p(x) are integers
   (b) All roots of p(x) are integers
   (c) P(0) = -1
   (d) P(5) = 2304

Solution:

Since coefficients and roots are integers, $p(x)=a(x-\alpha_1)^{r_1}(x-\alpha_2)^{r_2}...(x-\alpha_m)^{r_m}$ where a, α_1, α_2, ..., α_m are integers and $r_1, r_2, \ldots, r_m$ are positive integers.

$p(0) = -1 = a(-\alpha_1)^{r_1}(-\alpha_2)^{r_2} \ldots (-\alpha_m)^{r_m}$. Hence a, $\alpha_1, \alpha_2, \ldots, \alpha_m$ are all ± 1.

Therefore, $p(x) = a(x-1)^m(x+1)^n$.

Case a = 1:
$p(0) = (-1)^m = -1 \Longrightarrow$ m is odd
$p(5) = 4^m 6^n = 2^{2m+n} 3^n = 2304 = 2^8.3^2 \Longrightarrow$ n =2 and m = 3
Thus, $p(x) = (x-1)^3 (x+1)^2$

Case a = -1:
$p(0) = -(-1)^m = -1 \Longrightarrow$ m is even
But from $p(5) = 4^m 6^n = 2^{2m+n} 3^n = 2304 = 2^8.3^2$ we get m is odd. Not possible.
$p(x) = (x-1)^3 (x+1)^2$ is the only solution.

Alternate solution

Let $p(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n$.

Since roots are integers the leading coefficient is +1 or -1.
Take $a_0 = 1$.
Then $p(5) = 5^n + a_1 5^{n-1} + \ldots + a_{n-1} 5 - 1 = 2304$

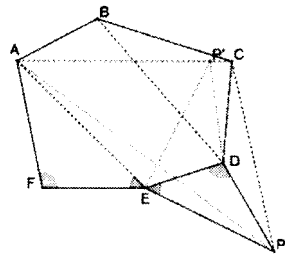
$$5^n + a_1 5^{n-1} + \ldots + a_{n-1} 5 = 2305$$

$$5^{n-1} + a_1 5^{n-2} + \ldots + a_{n-1} = 461$$

Hence $a_{n-1}=1$
$5^{n-2} + a_1 5^{n-3} + \ldots + a_{n-2} 5 = 92.$

Hence
$a_{n-2} = 2.$
$5^{n-3} + a_1 5^{n-4} + \ldots + a_{n-3} = 18.$ The only solution is
$25 - 5 - 2 = 18.$
This gives $n - 3 = 2$ is $n = 5$.
$\therefore$ The polynomial is $x^5 - x^4 - 2x^3 + 2x^2 + x - 1$

2. ABCDEF is a convex hexagon with $\angle B + \angle D + \angle F = 360^0$ and AB.CD.EF = BC.DE.FA. Show that BC.DF.EA = EF.AC.BD



Solution:

Take P so that ΔAFE and ΔPDE are similar. [So, take $\angle PDE = \angle AFE$ and $\angle PED = \angle AEF$. Note that you need to take the right point – P_1 fails below.]

So, $\frac{EF}{ED} = \frac{EA}{EP}$.

Also $\angle DEF = \angle DEA + \angle AEF = \angle DEA + \angle PED = \angle PEA$ (this is where we need P on the opposite side of DE to A).

Hence triangles DEF and PEA are similar. So $\frac{FD}{EF} = \frac{AP}{EA}$ ....(*)

Triangles AFE, PDE being similar also gives $\frac{FA}{EF} = \frac{DP}{ED}$.

So, $\frac{AB}{BC} = \frac{DE}{CD}\frac{FA}{EF}$ (given) $= \frac{DE}{CD}\frac{DP}{ED} = \frac{PD}{CD}$.

But $\angle CDE + \angle EDP + \angle PDC = 360^0$, or $\angle D + \angle EFA + \angle PDC = 360^0$.

We are given that $\angle B + \angle D + \angle F = 360^0$, so, $\angle PDC = \angle ABC$.

Hence triangles ABC and PDC are similar.

So, $\frac{CB}{CD} = \frac{CA}{CP}$ and $\angle BCD = \angle ACD + \angle BCA = \angle ACD + \angle DCP = \angle ACP$.

So, triangles BCD and ACP are similar.

Hence, $\frac{BC}{BD} = \frac{AC}{AP}$.

Multiplying by (*) gives $\frac{BC}{BD}.\frac{FD}{EF} = \frac{AC}{EA}$, which is the required equality.

3. a, b $\in$ N such that $\left(\frac{a+1}{b} + \frac{b+1}{a}\right) \in N$. Prove that HCF (a, b) $\leq \sqrt{a+b}$.

Solution:

Let h be the HCF(a, b) and let a = hm and b = hn where HCF(m, n) = 1.

Given that

$$\left(\frac{a+1}{b}+\frac{b+1}{a}\right)\in N$$

$$\Rightarrow \frac{a^2+b^2+a+b}{ab}\in N$$

$$\Rightarrow \frac{(a+b)^2-2ab+a+b}{ab}\in N$$

$$\Rightarrow \frac{(a+b)^2+a+b}{ab}-2\in N$$

$$\Rightarrow \frac{(a+b)(a+b+1)}{ab}\in N$$

$$\Rightarrow \frac{(hm+hn)(hm+hn+1)}{h^2mn}\in N$$

$$\Rightarrow \frac{(m+n)(hm+hm+1)}{hmn}\in N$$

We have two cases.

Case 1: $h \neq 1$

If $h \neq 1$, then $h \nmid (hm + hn + 1)$.

Hence $h \mid (m + n) \Rightarrow h^2 \mid (hm + hn) = a + b$.

Therefore $h^2 \leq (a + b)$ and hence $h = HCF(a, b) \leq \sqrt{a+b}$.

Case 2: $h = 1$

If $h = 1$, then $h = 1 \leq \sqrt{a+b}$, since $\sqrt{a+b} \geq \sqrt{2}$

Hence the result is true.

4. Prove that if $a_1, a_2, \ldots, a_n$ are real numbers, then

$$\sqrt[3]{a_1^3 + a_2^3 + \ldots + a_n^3} \leq \sqrt{a_1^2 + a_2^2 + \ldots a_n^2}$$

Solution:

Since the right hand side is positive the inequality is satisfied if the LHS is negative. WLOG let us assume that all a_k, k = 1, 2, …, n are ≥ 0.

If $0 \leq x \leq 1$, then $x^{3/2} \leq x$, and equality holds when x = 0 or x = 1. The given inequality is true as an equality if all the a_k's are zeroes. Assume that at least one of the numbers a_k is non-zero.

Let $x_k = \dfrac{a_k^2}{a_1^2 + a_2^2 + \ldots + a_n^2}$. Then $0 \leq x_k \leq 1$, and by the above quoted inequality,

we get,

$$\sum_{k=1}^{n} \left(\frac{a_k^2}{a_1^2 + a_2^2 + \ldots + a_n^2} \right)^{3/2} \leq \sum_{k=1}^{n} \left(\frac{a_k^2}{a_1^2 + a_2^2 + \ldots + a_n^2} \right) = 1$$

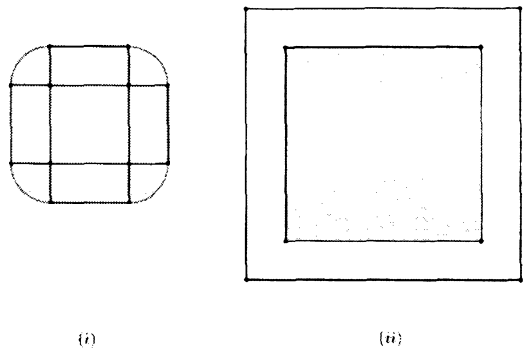
Hence,

$\sum_{k=1}^{n} a_k^3 \leq \left(\sum_{k=1}^{n} a_k^2 \right)^{3/2}$ which is what we need to prove.

For equality, exactly one of the x_k has to be one and the others zero. This means exactly one of the a_k's is positive and the others zero.

5. From a 90 × 90 square cloth, a sloppy tailor cuts 2021 square patches of side 1. Prove that no matter how he performs the cutting, he can still cut a circular patch of diameter 1 from the remaining material.

Solution:



(i) (ii)

Figure 1

Consider a square of side 1. The region of points that lie at a distance $\frac{1}{2}$ from any point of the square is shown in Figure 1 (i). If we take a point P outside this region and draw a circle of diameter 1, with centre at P , then the circle will not intersect the square.

Similarly, the region of points that lie inside the given square and are at a distance of at least $\frac{1}{2}$ from the sides of the square is shown in Figure 1 (ii). If we take any point P in this region, a circle with centre at P and diameter 1 will lie inside the square and will not intersect any side of the square. The total area covered by the 2021 regions of the type shown in Figure 1 (i) is

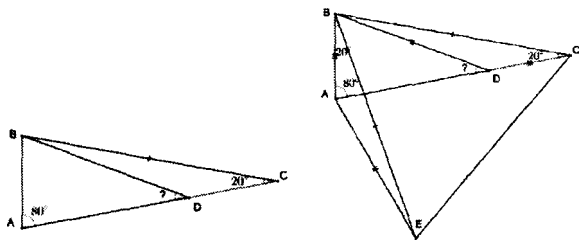
$$2021\times\left(1+4\times 0.5\times 1+\frac{\pi}{4}\right)=2021\times\left(3+\frac{\pi}{4}\right)$$

Since $\pi < 3.2$, this area is at the most $2021\times 3.8 = 7679.8 < 7921 = 89^2$.

Thus the 2021 regions will not cover the entire shaded region shown in Figure 1 (ii). Thus, we can find a point P which is at a

distance of more than $\frac{1}{2}$ from all the squares and also from the four boundaries. A circle with diameter 1 and centre at P is the required circle.

6. ABC is isosceles with vertical angle $\angle C = 20^0$. Point D on AC is such that CD = AB. Join BD. Find $\angle BDA$.



Solution: Construct Δ ABE congruent to Δ DCB. $\angle ABE = 20^0$. Hence $\angle EBC = 60^0$ and BE = BC. Therefore Δ BCE is equilateral.

Hence AC = BC = CE. Therefore $\angle ACE = \angle BCE - \angle BCA = 60^0 - 20^0 = 40^0$. Therefore, $\angle CAE = \angle CEA = 70^0$. Thus $\angle AEB = 180 - 20 - 80 - 70 = 10^0 = \angle DBC$. Hence, $\angle BDA = \angle DBC + \angle DCB = 10 + 20 = 30^0$.

7. A positive integer N is said to be good if its binary representation does not contain three equal consecutive digits. For example, the numbers 4, 5, 21 are good since, their binary representations are 100, 101, 10101 respectively while 7 is not good since its binary representation 111 contains three consecutive equal digits. Find the number of good integers N such that $1 \leq N \leq 2^{18}$.

Solution:

Suppose that the binary representation of N has k digits. We first count the number of good integers for k = 1, 2, 3, 4.

k	Good integers	Count
1	1	1
2	10, 11	2
3	100, 101, 110	3
4	1001, 1010, 1011, 1100, 1101	5

From the table, we observe the following pattern: If a_k denotes the number of good integers with k binary digits, then $a_k = a_{k-1} + a_{k-2}$. We will prove that this is true for all k .

Let us denote by x_k, y_k, z_k, u_k the number of integers with k digits in the binary representation and ending in 00, 01, 10, 11, respectively. Then, it is easy to see that

$x_k = z_{k-1}$; $y_k = x_{k-1} + z_{k-1}$; $z_k = u_{k-1} + y_{k-1}$; $u_k = y_{k-1}$

Clearly, $a_k = x_k + y_k + z_k + u_k$.

Also, $y_k + z_k = a_{k-1}$ and $x_k + u_k = z_{k-1} + y_{k-1} = a_{k-2}$. Hence
$a_k = x_k + y_k + z_k + u_k = a_{k-1} + a_{k-2}$

This proves our claim.

Since $a_3 = 3$; $a_4 = 5$, it follows that a_k for $k \leq 18$ are 1, 2, 3, 5, 8, 13, 21, 34, 55,
89, 144, 233, 377, 610, 987, 1597, 2584, 4181

The number of good integers $\leq 2^{18}$, is therefore the sum of the above 20 numbers and this is easily computed as 10944.

8. Find all quadruples (a, b, c, d) of positive integers such that

$$\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)\left(1+\frac{1}{d}\right)=5$$

Solution:

Without loss of generality assume a ≥ b ≥ c ≥ d. Then, $\left(1+\frac{1}{d}\right)^4 \geq 5$ which implies d ≤ 2.

If d = 1, then $\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)=\frac{10}{3}$

By a similar argument we can prove that a = b = c = 2. This does not satisfy the required condition.

If d = 1, then $\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)=\frac{5}{2}$

This implies $\left(1+\frac{1}{c}\right)^3 \geq \frac{5}{2}$. Hence c ≤ 2.

If **c = 1**, then

$$\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)=\frac{5}{4}$$

$$\Rightarrow 4(a+1)(b+1)=5ab$$

$$\Rightarrow ab-4a-4b=4$$

$$\Rightarrow (a-4)(b-4)=20$$

$$\Rightarrow (a,b)=(24,5),(14,6),(9,8)$$

If c = 2, then

$$\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)=\frac{5}{3}$$

$$\Rightarrow 3(a+1)(b+1)=5ab$$

$$\Rightarrow 2ab-3a-3b=3$$

$$\Rightarrow 4ab-6a-6b=6$$

$$\Rightarrow 4ab-6a-6b+9=6+9$$

$$\Rightarrow (2a-3)(2b-3)=15$$

$$\Rightarrow (a,b)=(9,2),(4,3)$$

Thus, we have 5 solutions:

(24, 5, 1, 1); (14, 6, 1, 1); (9, 8, 1, 1); (9, 2, 2, 1); (4, 3, 2, 1)

and their permutations.

ARYABHATTA CONTEST
NMTC at SENIOR LEVEL

1. Find the number of positive integers n less than 1000 for which there exists a positive real number x such that $n = x\lfloor x \rfloor$, where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

Solutions:

We begin our solution by first noticing that x cannot be irrational because the product of a rational number and an irrational number is irrational, but n is an integer. Therefore, x must be rational. Let $x = a + \frac{b}{c}$ where a, b, c are non-negative integers and $0 \le b < c$.

Then $n = \left(a + \frac{b}{c}\right)\left\lfloor a + \frac{b}{c} \right\rfloor = \left(a + \frac{b}{c}\right) \times a = a^2 + \frac{ab}{c}$

Now we need to determine a, b, c such that $\frac{ab}{c}$ is an integer. We can do so by finding values of n for value of a.

For $a = 0$, there is no possibility since n is a positive integer.

For $a = 1$, $b, c = 0, 1$. For $a = 2$, b, c is either 0, 2 or 1, 2. Similarly, for $a = 3$, b, c is either 0, 3 or 1, 3 or 2, 3. This pattern continues up to $n = 31$. If $a = 32$ then $n > 1000$. However, if $a = 31$, the largest possible value of x is $31 + \left\lfloor \frac{30}{31} \right\rfloor = 31$ for which n is still less than 1000. Therefore the

number of positive integer values of n must be $1+2+3+\cdots+31=\frac{31\times32}{2}=496$.

2. At a certain university, the division of mathematical sciences consists of the departments of mathematics, statistics, and computer science. There are two male and two female professors in each department. A committee of six professors is to contain three men and three women and must also contain two professors from each of the three departments. Find the number of possible committees that can be formed subject to these requirements.

Solutions:

There are two cases:

Case 1: One man and one woman is chosen from each department.

Case 2: Two men are chosen from one department, two women are chosen from another department, and one man and one woman are chosen from the third department.

For the first case, in each department there are (2 choose 1) × (2 choose 1) = 4 ways to choose one man and one woman. Thus there are $4^3 = 64$ total possibilities conforming to case 1.

For the second case, there is only (2 choose 1) = 1 way to choose two professors of the same gender from a department, and again there are ways to choose one man and one woman. Thus there are $1\times1\times4 = 4$ ways to choose two men from one department, two women from another department, and one man and one woman from the third department. However, there are $3! = 6$ different department orders, so the total number of possibilities conforming to case 2 is $4\times6 = 24$. Summing these two values yields the required answer $64 + 24 = 88$.

3. The degree measures of the angles in a convex 18-sided polygon form an increasing arithmetic sequence with integer values. Find the degree measure of the smallest angle.

Solutions:

The average angle in an 18-gon is 160°. In an arithmetic sequence the average is the same as the median, so the middle two terms of the sequence average to 160°. Thus for some positive (the sequence is increasing and thus non-constant) integer d, the middle two terms are $(160-d)^\circ$ and $(160+d)^\circ$. Since the step is $2d$, the last term of the sequence is $(160+17d)^\circ$, which must be less than 180°, since the polygon is convex. This gives $17d<20$, so the only suitable positive integer d is 1. The first term is then $(160-17)^\circ = 143^\circ$.

4. Let M_n be the $n\times n$ matrix with entries as follows: For $1\le i\le n, m_{i,i}=10, 1\le i\le n-1, m_{i,i+1}=m_{i+1,i}=3$, all other entries in M_n are zero. Let D_n be the determinant of M_n. Find the value of $\sum_{n=1}^{\infty}\frac{1}{8D_n+1}$.

Solutions:

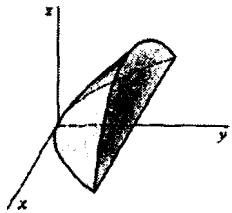
We can calculate the first few determinants to be 10, 91, 820, 7381. These satisfy the recursive relation $D_n=9D_{n-1}+1$. The values of $8D_n+1$ are 81, 729, 6561, 59049. The sum is then given by

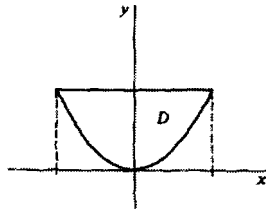
$$\sum_{n=1}^{\infty}\frac{1}{8D_n+1}=\frac{1}{81}+\frac{1}{729}+\frac{1}{6561}+\frac{1}{59049}+\cdots=\frac{1}{81}\left(\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n\right)=\frac{1}{81}\times\frac{9}{8}=\frac{1}{72}$$

5. Find the volume of the solid bounded by the surfaces $z=0, z=y-x^2, y=1$.

Solutions:

We will first draw the surface as in left side picture and then integrate over the region D given in right side to obtain its volume.





If V is the required volume then

$$V=\iint_D (y-x^2)dA=\int_{x=-1}^{1}\int_{y=x^2}^{1}(y-x^2)dydx=\int_{x=-1}^{1}\left(\frac{1}{2}-x^2+\frac{x^4}{2}\right)dx=\frac{8}{15}$$

6. Let m be a positive real number. Define the sequence $\{x_n\}$ by $x_{n+1}=m+x_n^{\,2}$ for all $n\geq 0, x_0=0$. Determine the necessary and sufficient condition such that the sequence $\{x_n\}$ is convergent.

Solutions:

Let the given sequence x_n possess limit L. Then from the definition of terms of the given sequence, we have $L = m + L^2$ from which we get a quadratic equation $L^2 - L + m = 0$. We get

$L = \frac{1 \pm \sqrt{1-4m}}{2}$. Hence if the sequence is convergent then necessarily, $m \leq \frac{1}{4}$.

Now if we assume that $m > 0$ but less than or equal to 1/4 then from the equation

$x_{n+1} - x_n = x_n^{\,2} - x_{n-1}^{\,2}$ it follows that the sequence x_n is increasing. However

$x_{n+1} = m + x_n^{\,2} < \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ provided $x_n < \frac{1}{2}$.

Hence the sequence is non-decreasing and bounded above and so it must be convergent provided

$0 < m \leq \frac{1}{4}$ and $x_n < \frac{1}{2}$. This will be the sufficient condition.

7. Find $\sum_{n=1}^{\infty} \frac{n}{n^4 + n^2 + 1}$

If a_n denotes the nth term of the given series then

$$a_n = \frac{n}{n^4 + n^2 + 1} = \frac{1/2}{n^2 - n + 1} - \frac{1/2}{n^2 + n + 1} = t_n - t_{n+1} \text{ where } t_n = \frac{1/2}{n^2 - n + 1}.$$

If S_N is sequence of partial sums of the given series then

$$S_N = \sum_{n=1}^{N} a_n = \sum_{n=1}^{N} (t_n - t_{n+1}) = t_1 - t_{N+1} = \frac{1}{2} - \frac{1/2}{N^2 + N + 1}$$

Thus we get

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{n}{n^4 + n^2 + 1} = \lim_{n \to \infty} (S_N) = \frac{1}{2} - 0 = \frac{1}{2}.$$

8. Nine delegates, three each from three different countries, randomly select chairs at a round table that seats nine people. Find the probability that each delegate sits next to at least one delegate from another country.

Use complementary probability and Principle of Inclusion-Exclusion. If we consider the delegates from each country to be indistinguishable and number the chairs, we have $\frac{9!}{(3!)^3} = 1680$ total ways to seat the candidates. Of these, there are $3 \times 9 \times \frac{6!}{(3!)^2} = 540$ ways to have the candidates of at least some one country sit together. Among these there are $3 \times 9 \times 4 = 108$ ways for candidates from two countries to each sit together. Finally, there are $9 \times 2 = 18$ ways for the candidates from all the countries to sit in three blocks (9 clockwise arrangements, and 9 counter-clockwise arrangements).

So, by Principle of Inclusion – Exclusion, the total count of unwanted arrangements is 540 – 108 + 18 = 450. Hence the required probability = $\frac{1680 - 450}{1680} = \frac{1230}{1680} = \frac{41}{56}$.

News to be proud of

The team AMTI is delighted to share the following news to the readers of the Mathematics Teachers.

The Government of India , every year, selects some distinguished personalities for the Padma Awards. This years list was realesed on 25 January 2023 by the Ministry of Home Affairs. The AMTI is proud of the following three personalities, who are included in this list.

(i) Shri Srinivas Varadhan, from USA, has been awarded 'Padma Vibhushan' Award in the field of science and Engineering 83 years old Shri Varadhan, born in Chennai, is at present working as a Mathematics Professor and E.J. Gould Professor at the courant Institute of Mathematical Sciences of New York University.15 years ago, he was awarded the Padma Bhushan and in 2007.He won the highly prestigious Abel Prize for his contributions to Probability Theory.

(ii) Prof. R.C. Gupta, former president of AMTI (1994) , has been awarded 'Padma Shri' for his contribution in the field of Literature and Education. He is well-known as a historian of mathematics. He did his Ph.D. in History of Mathematics from Ranchi University. He is a fellow of National Academy of sciences. He founded the magazine 'Ganit Bharati in 1979. He was awarded Kenneth O. May prize in 2009.

(iii) Dr. Sujatha Ramdorai (born in 1962) is awarded Padma Shri in the field of Science and Engineering. She is known for her work on non- cumulative Iwasawa Theory and Arithmatic of Algebraic varieties. She got her Ph.D. from TIFR. At present she is a professor of Mathematics and Canada Research Chair at University of British Columbia, Canada. She is a receipant of ICTP Ramanujan Prize and Shantiswaroop Bhatnagar Award among the others.

ASSOCIATION ACTIVITIES

A Report of the on-line Group Meeting held on **9th November 2020**, through **Zoom App**, commencing at 3.00 pm.

The zoom meeting was attended by
Sri R. Athamaraman,
Dr. M. Palanivasan,
Sri Palani Natarajan,
Sri Maruthi Natarajan,
Sri Balaji and Smt. Aruna.

The General Secretary requested Sri Palani Natarajan to send the user ID and password for our AMTI website immediately. Sri Maruthi Natarajan accepted to finalize the application form and the payment gateway process. Sri R. Athmaraman told Sri Maruthi Natarajan to speed up the process and complete the entire process by last week of November 2020. The General Secretary requested Sri Maruthi Natarajan to reconsider and reduce the amount quoted by him per student. He said that he would send a revised quotation for the same. The meeting concluded with vote of thanks by General Secretary,
Dr. M. Palanivasan.

ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

Minutes of the GENERAL BODY meeting held on 29.12.2020 via ZOOM at 5p.m. 31 members were present.

1. The meeting commenced with the prayer by Mr. S.R.Santhanam. Our president Prof. I.K.Rana preside over the meeting and welcome the gathering.

2. Condolence resolutions were passed for our former office bearer Mr. P.V.Arunachalam from Tirupathi and our life member Mr. Gotkhindikar Dilip from Nasik.

3. Minutes of the last General body meeting was then read by General Secretary Dr. M.Palanivasan with compliance report or inability to take some steps. It's was adopted unanimously.

4. Audited account for the year 2019-20 was perused by the member. Sri. S.R.Santhanam raised some clarification about donation and it was cleared. The audited accounts were adopted unanimously.

5. We have received some registrations for our summer workshop. Due to covid-19, we could not conduct the summer workshops. The amounts were refunded. Mathematics Teacher -55 Volume 1&2 and 3&4 were printed and dispatched. Mathematics Teacher -56 volume 1&2 was ready but could not be printed because of Covid – 19. Junior Mathematician March 2020 and September 2020 issues were ready but could not be printed. Both Mathematics teacher and Junior Mathematician will be printed and it will be dispatched to the members. e-copies will be sent to the requested members. Our patron Sri. R.Athmaraman suggested to send Junior Mathematician to all life members. We have conducted one popular lecture in December 2019.Some repair works were carried out in our project office.

6. Due to Covid-19, this year NMTC – 52 will be conducted through online mode. We have made an agreement with M/S. Maruthi Computers Ltd Chennai. They will take care

of First level exam and Second level will be conducted through paper, pen mode in their respective schools.

7. Due to Covid 19, our 55th Conference could not be conducted in December 2020.Our conference Secretary made some arrangements with Manipal University. The General Body members unanimously decided to postpone the 55th Conference.

8. In any other matters, Dr. S.R. Santhanam proposed Mr. Lakshmi Narayanan as an Executive Committee Member and seconded by Mr. Bhas Bamre. It's was approved by the General Body. Mrs. Vishakha Manoj Kabra requested to conduct the Summer Workshop through online and Our President also suggested to conducted monthly online workshop.

9. Mr. P.Ramesh proposed vote of thanks.

E C MEETING ON 26.6.2021 Report

Our AMTI E.C Meeting was held on 26.6.2021 at 5p.m via zoom 18 members were present. Our President Prof. I.K. Rana presides over the meeting. The following members were present

Prof. I.K Rana
Athmaraman. R. Sri
Mahadevan. M. Sri
Sundaramurthy.V. Sri
Santhi. R. Smt
Lakshmi Narayanan Sri
Bhas Vijay Bhamre Sri
Ramesh. P
Swaminathan . P.V
Pandurangan. J. Prof
Palanivasan. M. Dr
Hemalatha Thiagarajan. Mrs
Santhanam. S.R. Sri
Bakthavatchalu. P. Dr.
Vijayakumar. A. Dr
Jayapriya. S. Mrs.
Kumaraswamy. M. Dr
Muralidharan. S. Dr

The meeting started with a prayer song by our Conference Secretary Dr. S.R. Santhanam. Our General Secretary Dr. M. Palanivasan welcomed the members.

Talent Exam. Secretary Dr. Hemalatha.T presented a detailed report on 52nd NMTC 2020-21. She has explained the difficulties in that online exam. Regarding the NMTC 2021-22 Dr. S.R. Santhanam suggested to postpone the exam because of Covid problem. But our patron Sri R. Athmaraman requested not to postpone the exam. Dr. Hemalatha.T suggested to increase the fees for NMTC 2021-22. But Sri R. Athmaraman explained the motto of our AMTI to serve more number of poor students. Prof. I.K. Rana suggested to conduct the NMTC online exam in different dates and also suggested to form a committee to discuss the modalities of conducting NMTC Exam.

Regarding our AMTI conference Prof. I.K. Rana requested to tie up with TIME Organization to conduct the conference. Dr. S.R. Santhanam suggested to conduct our AMTI conference separately. Our President suggested to form academic and organizing committee for the conference. Prof. R. Vijayakumar has accepted to be a member of the academic committee.

Our General Secretary explained the financial position from April 2020 to June 2021. Since there was no income in the last year our President suggested to conduct more online workshop.

Status reports of MT and JM were discussed. Because of Covid problems, the Executive Member have suggested not to print "The Mathematics Teachers" and "Junior Mathematician" and requested to send the soft copies to the members through mail. Our workshop Secretary Dr. P. Bhakthavatchalu accepted to arrange more workshops. Our Popular Lecture Secretary Dr. M. Kumarasamy accepted to arrange a Popular Lecture on July 2021 and periodically.

Our General Secretary informed that Dr. Hemalatha.T wanted to relieve from Talent Exam Secretary post. Dr. R. Shanthi has

accepted to take charge of the Talent Exam Secretary. Hence Dr. Hemalatha.T was relieved form the Talent Exam Secretary. Dr. S. Muralidharan wanted to relieve form the Editor, "The Mathematics Teachers" and the Executive Committee requested our General Secretary to find a suitable alternate.

In order to increase the income, Shri Lakshmi Narayanan suggested to increase the NMTC fees slightly. Dr. S.R. Santhanam suggested to decrease the office expenses. Shri R. Athmaraman suggested to form a committee to minimize the office expenses and revise the salary for the staff. It was unanimously accepted.

Shri. P. Ramesh proposed vote of thanks.

Regret: In the Vol.56 (3&4) of 'The Mathematics Teacher', an article on Page.29 titled "Some musings on Primes" by K. Srinivas is published. It is based on the " Prof. R.C. Gupta Endowment Lecture" given at the 54th Annual Conference of AMTI. We feel sorry for the mistake.

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